The Glasses are Tinted: Self-Confidence and Poverty Trap

Dyotona Dasgupta Anuradha Saha





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July 2020

Abstract

In an overlapping generations model, this paper looks at the effects of behavioral anomalies on human capital investments and skill distributions. In this model education is necessary but not sufficient to get a skilled job. There are three types of agents: uneducated-unskilled, educated-unskilled, educated-skilled. Behavioral anomalies are such that adults underestimate the probability of intergenerational mobility. Uneducated-unskilled are imprisoned in a behavioral trap – they do not believe that an educated child from their community would get a skilled work, so they never invest. Educated parents suffer from a behavioral bias – skilled ones overestimate the chances of their educated children getting a skilled job while unskilled ones underestimate. The educated parents may over or under invest in comparison to the case where they have correct beliefs. Behavioral trap almost always causes poverty trap and also gives rise to multiple steady states, which can be ranked in terms of inequality. Depending on the degree of behavioral bias of the educated parents, steady state inequality could be lower or higher than that when they have correct beliefs. However, even in a less unequal society the opportunity to earn higher income is limited to only a fraction of population. Behavioral bias may lead to multiple equilibria and even induce (poorer) educated-unskilled adults to invest with higher probability than (richer) skilled persons.

Keywords: Human Capital Investment, Behavioral Bias, Poverty Trap, Behavioral Trap **JEL Codes**: I3, D9, E2

^{*}We acknowledge useful comments from Mausumi Das, Sayantan Ghosal, Parikshit Ghosh, Ratul Lahkar, Dilip Mookherjee, Prabal Roy Chowdhury, and Arunava Sen and participants of Delhi Theory Workshop at Indian Statistical Institute and seminar participants at Delhi School of Economics. Dyotona Dasgupta acknowledges postdoctoral fellowship funding jointly provided by the Centre for Development Economics, Delhi School of Economics and the Institute of Economic Growth during 2019-20. All errors are our own.

[†]E-mail: dasgupta.dyotona@gmail.com

[‡]Ashoka University. E-mail: anuradha.saha@ashoka.edu.in.

1 Introduction

This paper studies the implications of behavioral constraints on human capital investment and the resultant impacts on skill distribution and welfare of an economy. We provide an explanation of how decisions based on own experience can give rise to poverty trap, affect inequality, and distort incentives to invest in human capital.

Traditional literature has focused on capital market imperfections or technological indivisibilities (Banerjee and Newman (1993), Galor and Zeira (1993), Mookherjee and Ray (2002a), more in Related Literature section) to explain persistent inequality and poverty trap. While external constraints play an important role, internal constraints can also pull-down a a segment of the society into a poverty trap. Recent literature (see Besley (2017), Genicot and Ray (2017) for example) looks at behavioral aspects. The literature (see Bénabou and Tirole (2016) for an overview) mostly focuses on beliefs that reinforce a positive self-image. In our model, biases generate through socio-economic background and thus addresses both positive, and negative self-image.

Let us consider women in politics in South Asian economies. By the late 1990s, South Asian countries, such as India, Pakistan, and Bangladesh, had seen their first female Prime Ministers. These women came from influential families and broke the political glass ceiling in their respective countries. However, this major breakthrough has not converted into a higher representation of women in the national Parliament, where a privileged background is still an important criterion for women to win popular mandates. In contrast, men from all walks of life have had considerable success in the field. Why do female politicians come from a specific background while such restrictions are less so for male politicians? Apart from social, cultural, and financial constraints, behavioral constraints also play a role. If there are so few successful female politicians, especially from the underprivileged social background, it does not inspire confidence in women to choose this career path. We such behavioral constraints for women more starkly in careers such as STEM research (reference), film direction (reference), or military forces (reference).

To this end, we construct an overlapping generations model where adults differ in terms of their education, and jobs – skilled or unskilled. They derive utility from their own consumption and the perceived expected income of their children. To capture the effect of behavioral anomaly solely, we abstract from any bequest motive. Income of a skilled worker is higher than that of an unskilled worker. But, costly education is necessary, though not sufficient, for getting a skilled job – an uneducated individual works as an unskilled worker with certainty whereas an educated person gets a skilled job with an exogenously given probability. This probability is the same for all children implying no intrinsic difference among them. Based on their education and jobs, adults can be classified into three groups – not educated and unskilled, educated and unskilled, educated and skilled. These groups are endogenously formed and can change every period. Each adult decides whether to invest in her child's education or not. The expected benefit from that investment depends on the perceived probability of her educated child becoming a skilled worker and the income that she would earn as a skilled worker.

We say that the agents are biased when they form a belief about the probability of success based on their socio-economic circumstance. In the model, parents underestimate the probability with which an educated child from her group moves to another group. This is because adults from socially distant groups identify less with each other. We capture this through "degree of association". Lower the degree of association, more biased is the perceived probability of success. An unskilled worker underestimates the probability with which a child from her group, upon getting education, becomes a skilled worker. Due to higher social distance, a not educated and unskilled worker underestimates more than an educated and unskilled worker. Conversely, an educated and skilled worker under estimates the probability of her educated child becoming an unskilled worker. Alternatively, we may say the vision of a not educated and unskilled worker is very tinted, education partially clears it. Here, just as in classic Spence (1973), education has no intrinsic value, but we may say education makes people more open to exploring economic opportunities. All agents are non-Bayesian and there is no convergence of beliefs. However, given their beliefs, each parent correctly calculates the equilibrium mass of skilled worker and their income.

We characterize the equilibria and analyze the dynamics and steady states of the economy. An equilibrium specifies the probabilities of investment of each type of worker, such that no one has any incentive to deviate unilaterally and their decision is consistent with their beliefs. We study the implications of behavioral bias by comparing the results with the case without such biases. The initial income of a skilled worker is the state variable. Investment decisions crucially depend on the state variable, the degree of child affinity and the degree of association characterize the equilibria, dynamics and the steady states. Before we discuss the results, a word about the degree of child affinity is in order. It is the weight a parent places on the utility from her child's expected income relative to the utility from their own consumption. The parameter is non-negative, time-independent, and common for all parents in an economy. As noted in Boca et al. (2014) children may be valued more or less than parents' own consumptions, correspondingly the child affinity parameter may be a fraction or greater than unity. (See Browning et al. (2014) pp. 106-120 for further discussion).

In the benchmark case where there is no behavioral anomaly, we find that whenever unskilled workers invest with a positive probability all skilled workers invest with certainty. The reason is as follows. First, without any behavioral anomaly, the expected benefit from investment is the same for all parents. Therefore, for a given degree of child affinity, the optimal investment decisions differ only due to the difference in income. Second, the utility cost of investment is lower for a skilled worker as her income is higher than that of an unskilled worker and the utility function is assumed to be concave. Due to this, the equilibrium, at any given degree of child affinity and initial skilled income, is unique. The optimal probability of investment for all parents weakly increases with the state variable and higher child affinity.

We find that economic outcomes have distinct properties as per three child affinity ranges – low, moderate, and high. At high child affinity all parents invest in their children's education for all state variable. Everyone is educated in the economy. For moderate child affinity, skilled parents, always, invest with a positive probability while unskilled parents invest only when the state variable is high. In the steady state, all skilled workers invest with certainty and unskilled workers invest with a positive probability. Finally, with low child affinity, unskilled workers never invest in their children's education. Over time, the economy asymptotes to a s steady state where no parent invests in their children's education. The steady states can be ranked in terms of inequality, the difference between the income of a skilled worker and that of an unskilled worker. Higher the skilled income, higher is the income inequality in the economy. Thus, the steady state income inequality increases from

zero at low child affinity to a positive constant at high child affinity.

Poverty trap is defined to be a situation where there exists a positive mass of families that never becomes rich. Alternatively, we say there is no poverty trap in an economy if at any period the probability with which a family becomes rich, that is the adult member works as a skilled worker is positive. We find, in the benchmark case, there is a poverty trap only when the degree of child affinity is low. When it is not low, at the steady state, the probability with which an adult from any family works as a skilled worker is positive. That probability weakly decreases with decrease in the degree of child affinity – it remains constant when the degree of child affinity is high and strictly decreases when that is moderate.

Behavioral anomaly significantly affects the economy. First, we consider when only not educated workers are imprisoned in a behavioral trap – they do not believe that an educated child from their group would ever be able to get a skilled job. This implies they never invest in education. The educated parents take this into account while making their investment decision. Like the benchmark case, the skilled workers invest with certainty whenever the educated and unskilled workers invest with a positive probability. Thus, again, at any given parametric condition, the equilibrium is unique.

We also find, the educated and unskilled workers invest with (weakly) higher probabilities than in the benchmark case. Precisely, at any parametric condition where, in the benchmark case, the unskilled workers invest with a non-degenerate probability and the mass of not educated workers is positive, the educated and unskilled workers invest with a strictly higher probability. The reason is as follows. The production function in the skilled sector is assumed to be concave, hence, at any period income of a skilled worker is inversely related to the mass of skilled workers at that period. Thus, non-investment of not educated workers increase the benefit from investment which makes educated and unskilled workers invest with a higher probability. The probability with which skilled workers invest, remain unchanged, as when they invest with a probability less than 1, unskilled workers do not invest even without any bias, so non-investment of not educated and unskilled workers does not have any effect.

The economic outcomes are same for the low child affinity case. For moderate and high child affinity, there are now multiple steady states. This is because even when the income of a skilled worker is very high which implies the mass of not educated workers is large, they do not invest. The multiple steady states can be ranked on the basis of inequality. Higher the steady state income of a skilled worker, higher is the steady state inequality. The steady state inequality is at least as high as that in the benchmark case. They are equal at the 'least unequal steady state'. However, in behavioral trap, once a child does not get education, her family never gets the opportunity to earn the higher income. In the benchmark case, that opportunities and are stuck in a poverty trap. We find, in contrast to the benchmark, even when the child affinity is not low, there is almost always a poverty trap – when child affinity is high, there is a poverty trap whenever the economy has some uneducated workers and when the child affinity is moderate, there is always a poverty trap under behavioral trap.

Next, we analyze the case where educated workers are also biased and underestimate the probability of intergenerational mobility. Not educated and unskilled workers, like before, are in a behavioral trap, and thus, do not invest. The educated parents no longer have the same expected benefit from educational investment as the skilled ones are overconfident and the unskilled parents are underconfident. So, the skilled workers no longer invest with

certainty when educated and unskilled workers invest with a positive probability. There could be multiple equilibria when the degree of child affinity is not low and the degree of association with other group is lower than the proportion of skilled workers to educatedunskilled workers. In all other cases, there is a unique equilibrium.

We compare the investment probabilities with those at the benchmark case. An educated and unskilled worker underestimates the probability with which an educated child from her community would become a skilled worker. This implies that she under estimates the mass of skilled workers in the next period and over estimates their income. Thus, an educated and unskilled worker may under or overestimate the benefit from investment which makes her under or overinvest in comparison to the benchmark case. Similarly, for the skilled worker. In fact, the over investment of educated and unskilled workers may crowd out the investment of skilled workers. The unskilled and educated workers may even invest with a higher probability than skilled workers.

Again, due to the non-investment of the not educated workers, there are multiple steady states for moderate and high child affinity. Interestingly, when the degree of child affinity is moderate, due to over investment, the steady state income inequality could be lower than the benchmark case. However, even then the opportunity to earn higher income is limited to the educated families – once a family becomes not educated, it never gets the opportunity to earn higher income as a skilled worker. There is almost always a poverty trap in an economy. We compare the mass of families in a poverty trap when all workers have some behavioral anomaly vis-á-vis when only not educated and unskilled workers are in a behavioral trap – it can be higher or lower depending on the over or under investment of the educated workers.

In summary, experience based beliefs on intergenerational mobility may lead to over or under investment in human capital, and thus create a less or more unequal society. However, even in a less unequal society, internal constraint imprisons certain individuals from participating in opportunities which may lead to earning higher income. Behavioral anomalies give rise to poverty trap and affect income inequalities.

Section 2 discusses relation to existing literature. Section 3 sets up the general framework of the model. Section 4 studies the benchmark case where there is no behavioral anomaly. Section 5 addresses two types of behavioral anomaly – Subsection 5.1 analyzes the case where only not educated workers are under behavioral trap whereas Subsection 5.2 addresses the case where all types of workers are biased. Section 6 compares these cases and studies the welfare implications of behavioral anomaly. Proofs are collected in respective Appendices 8.

2 Related Literature

This paper is related to various strands of literature. First, that behavioral anomalies shape an individual's decisions are backed by relatively new but a long list of papers. Bordalo et al. (2016) discuss how stereotypes are used to form probability judgments. While stereotypes ease the mental cost of decision making in an uncertain environment, they also bring in biases. For example, boys are better in math and science compared to girls (Strauss (2013)), or that caste determines one's natural aptitude and personality traits in India (Srinivasan et al. (2016)). Even though stereotypes provide a kernel-of-truth, they are very context-dependent. For example, in a randomized experiment, when Hoff and Pandey (2014) ask low- and highcaste boys to solve mazes, their performance is similar when their castes are not revealed. However, on revealing caste in mixed-caste groups, the low-caste boys under-perform. This suggests that stereotypes can affect behavior in settings where it should not have any influence. Self-confidence depends on social background, gender, wealth. Deshpande and Newman (2007) find that graduating students from reserved (backwards) category have significantly lower occupational *expectations* than their non-reservation counterparts. On a gender paradigm, Barber and Odean (2001) find men are overconfident and invest more. However, Case and Shiller (2003) and Malmendier and Tate (2005), document that people are, in general, overconfident. There are series of paper by Bénabou and Tirole (2002, 2004, 2006) which relate self-confidence with effort or investment decisions.

There is widespread evidence that different types of people are prone to biases in different environment. For example, Carvalho et al. (2016) find that poor exhibit present-bias in intertemporal monetary decision just before their paydays. Ramiah et al. (2016) find that Australian corporate treasurers are prone to various behavioral biases (such as self-serving, high confidence, loss aversion and anchoring biases). These behavioral biases affect working capital decisions and thus could have significant effect on the performance of the firm. Benartzi and Thaler (2007) find that senior citizens follow heuristics to save for their retirement. Their naivety and passive behavior may dictate capital flowing into sub-optimal portfolios. The authors suggest low-cost interventions to rectify some of the adverse effects of these biases on retirement savings.

Second, our paper is also related to the literature on aspiration. Appadurai (2004), Ray (2006), Genicot and Ray (2017), Mookherjee et al. (2010) model aspiration as a target income where parents derive utility if their child earns more than the target income. In our paper, while there are no target incomes for children, but parents obtain utility from their children's expected incomes. In the behavioral model we find that when parents have sufficiently high child affinity, educated-unskilled parents may invest with a higher probability than skilled parents. In spite of their under confident beliefs, the educated-unskilled parents display eagerness and ambition for their children.

The closest two papers are Dalton et al. (2016) and Besley (2017). In a simple static model Dalton et al. (2016) show the interaction between aspirations and poverty. Given an aspiration target, agents put in effort to maximize their wealth and hence utility. However, there exists complementarities between initial wealth and effort which influences poor persons to put in low effort which in turn leads to lower aspirations. This two-way relationship between aspirations and effort provides a key policy implication that aspiration-enhancing policies can enhance the effectiveness of policies that poverty alone. We also find that behavioral trap encompasses several generations into a state of poverty.

Third, our paper discusses how does behavioral trap interact with a poverty trap. There are several papers such as Loury (1981), Galor and Zeira (1993), Banerjee and Newman (1993), Mookherjee and Ray (2002b), (2003) which explain inequality stemming from nonconvex costs or capital market imperfections. Historically, people have been treated differently on the basis of caste, religion, skin color. So, parents from such backgrounds may well take that into account, and rationally choose to invest less. On the other hand, employers may also not hire them rationally. This paper does not deal with these, instead, it addresses only internal constraint.

3 Model

3.1 The Firms

We consider a single good economy comprised of a continuum of individuals of size 1. The good can be produced using only one input – labor. Labor is of two types – skilled (L_{st}) and unskilled (L_{ut}) .¹ Let the production function of the skilled sector be AL_{st}^{ϕ} , where $0 < \phi < 1$, and $A \ge 1$ – the production function is strictly increasing and strictly concave. The production function of the unskilled sector is L_{ut} . At any period t, the profit functions of representative firms of the skilled and unskilled sectors, π_{st} and π_{ut} , are given by:

$$\pi_{st} = AL_{st}^{\phi} - w_{st}L_{st}, \qquad \text{and} \qquad \pi_{ut} = L_{ut} - w_{ut}L_{ut}$$

where w_{jt} denotes the wage rate of a worker of type $j, j = \{s, u\}$. Solving the profit maximization problems we get

$$w_{st} = A\phi L_{st}^{-(1-\phi)}, \quad \pi_{st} = (1-\phi)AL_{st}^{\phi}, \quad \text{and } w_{ut} = 1.$$
 (1)

The profit of the skilled sector is divided among the skilled workers. So, the income of a skilled worker is $m_{st} \equiv w_{st} + \pi_{st}/L_{st} = AL_{st}^{-(1-\phi)}$ and that of an unskilled worker is $m_{ut} = 1$.

Observation 1. A skilled worker earns (weakly) more than an unskilled worker.

We show this in Appendix A.1.

A skilled worker earns strictly more than an unskilled worker whenever A > 1 or the mass of unskilled workers is positive.²

3.2 The Households

In a discrete time framework, we build an overlapping generations model with no population growth. An individual lives for two periods: first as a child and later as an adult. Adults share a common degree of child affinity, δ – higher δ captures higher child affinity. A representative household consists of an adult and a child. The adult works, earns income, consumes, and, depending on her degree of child affinity, decides whether to invest in her child's education.³ Investment in education requires a fixed cost denoted by \bar{s} , where $\bar{s} \in (0, 1)$. Education is necessary but not sufficient for becoming a skilled worker – an educated individual, denoted by e, becomes a skilled worker with probability β whereas a not educated person, denoted by n, becomes an unskilled worker with certainty: $Pr(L_t = L_{st}|e) = \beta$ and $Pr(L_t = L_{st}|n) = 0$.

An adult derives utility from her own consumption and from her child's expected income earned in the next period. The utility of an adult of type ij where i denotes her education

¹In all notations, subscripts s and u designate skilled and unskilled workers, and subscript t denotes time.

²Since, $\beta < 1$, the mass of unskilled workers could be zero only at t = 0, when the economy starts with all skilled workers. Even in such an economy, from t = 1 onwards, the income of a skilled worker would be higher than that of an unskilled worker, with certainty.

³For simplicity, we assume that an individual consumes only in her adulthood.

 $i \in \{e, n\}$ and j denotes her skill $j \in \{s, u\}$ is

$$U_t^{ij}\left(c_t^{ij}, E\omega_{t+1}^{ij}\right) = \frac{\left(c_t^{ij}\right)^{\sigma}}{\sigma} + \delta \frac{\left(E\omega_{t+1}^{ij}\right)^{\sigma}}{\sigma}, \quad \sigma < 0$$

 c_t^{ij} denotes her consumption, and $E\omega_{t+1}^{ij}$ denotes the expected income of her child. Observe, the utility function is strictly increasing and strictly concave.

The investment decision on child's education is made based on the *perceived* expected income of an educated child. It depends on the probability of her becoming a skilled worker upon getting education and the income she earns as a skilled worker which further depends on the mass of the skilled workers at that time. A parent forms beliefs about this probability and based on that belief, the parent calculates the mass of skilled workers and their income in the next period. A parent's belief depends on her own experience.

In our model, there is no inherent difference in the probability of getting a skilled job across educated children of different parent types. These odds are the same for all educated children, i.e., this probability is independent of her parent's type (education and income). So, any type dependent belief captures the agent's cognitive limitation. This is the only behavioral anomaly we focus on. The agent otherwise is rational. *Given her belief about the probability of her child becoming a skilled worker*, she accurately calculates the mass of skilled workers in the next period and makes the investment decision accordingly.

Let us introduce some more notations. Let p_{t+1}^{ij} be the probability with which a parent of type ij believes that her educated child will become a skilled worker at t+1. Given her belief, a parent calculates the mass of skilled worker at t+1. To differentiate it from the actual mass of skilled worker at t+1, we say that a parent of type ij conjectures that the mass of skilled worker would be L_{st+1}^{ij} and their income ω_{st+1}^{ij} . Thus, the perceived expected income of her educated child would be $E\omega_{t+1}^{ij} = p_{t+1}^{ij}\omega_{st+1}^{ij} + (1-p_{t+1}^{ij})\omega_{ut+1}^{ij} = p_{t+1}^{ij}\omega_{st+1}^{ij} + 1-p_{t+1}^{ij}$. The expected income of an uneducated child is $E\omega_{t+1}^{ij} = 1$.

At any period t, a parent compares *perceived* expected utility from investing in her child's education with that from not investing and invests only when the former is (weakly) higher

 U_t^{ij} (from investing in child's education) $\geq U_t^{ij}$ (from not investing in child's education)

$$\Rightarrow \quad \frac{(m_{it} - \bar{s})^{\sigma}}{\sigma} + \delta \frac{[p_{t+1}^{ij} \omega_{t+1}^{ij} + (1 - p_{t+1}^{ij})]^{\sigma}}{\sigma} \ge \frac{m_{it}^{\sigma}}{\sigma} + \frac{\delta}{\sigma}$$
$$\Rightarrow \quad \delta \left[\frac{[p_{t+1}^{ij} \omega_{st+1}^{ij} + (1 - p_{t+1}^{ij})]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right] \ge \frac{m_{it}^{\sigma}}{\sigma} - \frac{(m_{it} - \bar{s})^{\sigma}}{\sigma}. \tag{2}$$

The left hand side of the above inequality is the *perceived* expected net benefit from investing in child's education whereas the right hand side is the utility cost of making that investment.

Observe, if \bar{s} were zero then all types of parents would have invested in their children's education irrespective of their beliefs and incomes. Alternatively, if \bar{s} were greater than 1, then no unskilled parent could have *afforded* to educate her child. The assumption $\bar{s} \in (0, 1)$ rules out these two uninteresting cases.

We characterize the equilibria and analyze the dynamics of an economy. An equilibrium, in our model, has two features:

- (a) Parents calculate the expected return from investment which must be consistent with their beliefs.
- (b) No parent has an incentive to deviate unilaterally.

Next, we start our analyses with a benchmark case – the parents do not have any behavioral anomaly, they believe that the probability with which an educated child becomes a skilled worker is β .

4 Benchmark Case

A worker of any type ij, believes that the probability of an educated child becoming a skilled worker is β , i.e. $p_{t+1}^{ij} = \beta \forall i, j$. In absence of any behavioral differences between parents, their optimal decisions differ only due to differences in their incomes.

Let, at any period t, the probability with which a worker of type j invests in her child's education be λ_{jt} . So, at period t+1, the mass of skilled worker and their income⁴ would be

$$L_{st+1} = \beta [\lambda_{st} L_{st} + \lambda_{ut} L_{ut}], \text{ and } m_{st+1} = A [\beta [\lambda_{st} L_{st} + \lambda_{ut} L_{ut}]]^{-(1-\phi)}.$$

At t, a worker of type j invests in her child's education with probability λ_{jt} if and only if

$$\delta\left[\frac{\left[\beta^{\phi}A[\lambda_{st}L_{st}+\lambda_{ut}L_{ut}]^{-(1-\phi)}+1-\beta\right]^{\sigma}}{\sigma}-\frac{1}{\sigma}\right] \ge \frac{m_{jt}^{\sigma}}{\sigma}-\frac{(m_{jt}-\bar{s})^{\sigma}}{\sigma}.$$
(3)

where $L_{ut} = 1 - L_{st}$ and the inequality binds for j^{th} type when $\lambda_{jt} \in (0, 1)$.

An equilibrium is denoted by $\langle \lambda_{ut}, \lambda_{st} \rangle$ which satisfies the features described in Subsection 3.2. Observe, here the equilibrium concept is Nash Equilibrium. Comparing the investment decisions of the skilled and unskilled workers, we find:

Lemma 1. Consider any equilibrium $\langle \lambda_{ut}, \lambda_{st} \rangle$

- (a) if an unskilled worker invests in her child's education with positive probability $(\lambda_{ut} > 0)$, then a skilled worker invests in her child's education with certainty $(\lambda_{st} = 1)$,
- (b) at any period t, the probability of investment of both types of workers (weakly) increase with increase in income of a skilled worker at that period.

We show this in Appendix A.2.

Intuitively, Part (a) is an immediate implication of our assumption of concave utility function. Due to this assumption, the utility cost of investment decreases with income of a worker. The benefit from investment is the same for all the parents. Hence, whenever an unskilled worker invests, a skilled worker with higher income (see Observation 1) invests with certainty.

For Part (b), we refer to equation (3). When income of skilled workers increase (i) the utility cost of investment for the skilled workers decreases whereas that of unskilled workers

⁴Observe here, in this benchmark case, $\omega_{st+1}^{ij} = m_{st+1}$.

remains the same, and (ii) the benefit from investment increases. While the reason for the former is the concavity of utility function, the intuition behind the latter is a bit subtle. The benefit from investment at any period t, increases with the probability of becoming a skilled worker, upon getting education and the income of a skilled worker at period t + 1. Now, the probability of becoming a skilled worker, upon getting education is a constant. We argue that the income of skilled workers of two consecutive periods are positively (non-negatively) related, keeping investment decisions the same. Therefore, the benefit from investment, at any period t, increases with the income of the skilled workers of that period.

To see this positive relationship between the income of skilled workers of two consecutive periods, keeping investment decisions the same. Consider, two economies with two different income of skilled workers m_{st}^1 and m_{st}^2 with $m_{st}^1 > m_{st}^2$ where investment decisions are the same, i.e. $\lambda_{st}^1 = \lambda_{st}^2 = \lambda_{st}$ and $\lambda_{ut}^1 = \lambda_{ut}^2 = \lambda_{ut}$. Then, we argue that m_{st+1}^1 is greater than m_{st+1}^2 . For that, first observe, income of the skilled workers is inversely related to the mass of skilled workers. So, the mass of skilled worker in Economy 1 is lower. Second, Part (a) of this observation tells us that $\lambda_{st} \ge \lambda_{ut}$, in fact, $\lambda_{st} = 1$, whenever $\lambda_{ut} > 0$. So, in Economy 2 higher mass of skilled workers would be lower in Economy 1 which implies m_{st+1}^1 would be higher than m_{st+1}^2 . Hence, the positive relationship.

The implications of the lemma are as follows. Part (a) implies that there can be five equilibria $\langle \lambda_{ut}, \lambda_{st} \rangle$ – three pure strategies and two mixed strategies. The pure strategy equilibria are: (i) both skilled and unskilled workers invest with certainty $\langle 1, 1 \rangle$, (ii) unskilled workers do not invest while skilled workers invest with certainty $\langle 0, 1 \rangle$, (iii) no worker invests $\langle 0, 0 \rangle$. The mixed strategy equilibria are (iv) unskilled workers invest with positive probability and skilled workers invest with certainty $\langle 0 < \lambda_{ut} < 1, 1 \rangle$, (v) unskilled workers do not invest while skilled workers invest with positive probability $\langle 0, 0 < \lambda_{st} < 1 \rangle$. Part (b) implies that the income of a skilled worker at any period is the state variable of that period.

The degree of child affinity⁵ plays an important role in parent's investment decision. Next, we define three thresholds of child affinity which will be useful in further analyses.

Definition 1. The degree of child affinity is

- (i) 'high' when $\delta \geq \overline{\delta}$, where $\overline{\delta} \equiv \frac{(1-\overline{s})^{\sigma}-1}{1-(A\beta^{\phi}+1-\beta)^{\sigma}}$,
- (ii) 'moderate' when $\delta \in [\underline{\delta}, \overline{\delta})$, where $\underline{\delta} \equiv (1 \overline{s})^{\sigma} 1$, or
- (iii) 'low' when $\delta < \underline{\delta}$.

Observation 2. $0 < \underline{\delta} < \overline{\delta}$.

Appendix A.3 proves the ranking.

Consider any equilibrium $\langle \lambda_{ut}, \lambda_{st} \rangle$. Given Lemma 1, we know when $\lambda_{ut} > 0$, then $\lambda_{st} = 1$. Based on this, for a given degree of child affinity, we define three thresholds of the state variable. We will find these thresholds helpful in further analyses.

⁵For brevity, we use child affinity and degree of child affinity interchangeably.

Definition 2. Let $\langle \lambda_{ut}, \lambda_{st} \rangle$ be an equilibrium at the state variable m_{st} . For a given degree of child affinity

- (i) $\underline{b}_s(\delta)$ is the maximum value of the state variable, at which the skilled workers do not invest, i.e. $\lambda_{st} > 0$ if and only if $m_{st} > \underline{b}_s(\delta)$.
- (ii) $b_s(\delta)$ is the minimum value of the state variable, at which skilled workers invest with certainty, i.e. $\lambda_{st} = 1$ if and only if $m_{st} \geq \bar{b}_s(\delta)$.
- (iii) $\underline{b}_u(\delta)$ is the maximum value of the state variable, at which unskilled workers do not invest, i.e. $\lambda_{ut} = 0$ if and only if $m_{st} \leq \underline{b}_u(\delta)$.

We provide the formal expressions of these thresholds of the state variable in Appendix A.4.

Observe from Lemma 1 and equation (3), at $\underline{b}_u(\delta)$, there is a unique equilibrium $\langle \lambda_{ut}, \lambda_{st} \rangle$ where $\lambda_{ut} = 0$ and $\lambda_{st} = 1$. Similarly, at the unique equilibrium at $\overline{b}_s(\delta)$, $\lambda_{ut} = 0$ and $\lambda_{st} = 1$. And finally, at the unique equilibrium at $\underline{b}_s(\delta)$, $\lambda_{ut} = 0$ and $\lambda_{st} = 0$. We cumulate the ranking and other features of these thresholds in the following lemma.

Lemma 2. Properties of the thresholds of the state variable

(i)
$$\underline{b}_s(\delta) < b_s(\delta) < \underline{b}_u(\delta)$$

- (*ii*) $\underline{b}_s(\delta)$, $\overline{b}_s(\delta)$ and $\underline{b}_u(\delta)$ are decreasing in δ .
- (iii) Suppose, the degree of child affinity is
 - (a) high, then $\underline{b}_s(\delta) < \overline{b}_s(\delta) < \underline{b}_u(\delta) < 1$,
 - (b) moderate, then $\underline{b}_s(\delta) \leq 1 < \overline{b}_s(\delta) < \underline{b}_u(\delta)$,
 - (c) low, then $\underline{b}_u(\delta) = \infty$ and $1 < \underline{b}_s(\delta) < \overline{b}_s(\delta) < \underline{b}_u(\delta)$.

We prove this lemma in Appendix A.5.

Intuitively, as the parents become more child loving, the benefit from investment increases. At the thresholds, the parents are indifferent, to make them that, thus, the thresholds have to adjust accordingly. As observed earlier, the benefit from investment increases with the state variable. And, with increase in the state variable, the utility cost of investment decreases for the skilled workers and remains constant for the unskilled workers. Hence, the thresholds of state variable must decrease with increase in the degree of child affinity, such that the parents remain indifferent. The ranking of the thresholds, directly follows from Lemma 1. At $\underline{b}_u(\delta)$, unskilled workers are indifferent between investing and not investing. So from Part (a) of that lemma, we know that the skilled workers must invest with certainty at that threshold. The skilled workers invest with certainty as long as m_{st} is no less than $\overline{b}_s(\delta)$. Thus, from Part (b) of that lemma, $\underline{b}_u(\delta)$ is greater than $\overline{b}_s(\delta)$. As m_{st} falls below that threshold, again, from Part (b) of that lemma, we know that skilled workers no longer invest with certainty. They invest with a positive probability as long as m_{st} is no less than $\underline{b}_s(\delta)$. Hence, $\underline{b}_s(\delta)$ is lower than $\overline{b}_s(\delta)$.

Given the parameters δ , σ , \bar{s} , β , and the state variable m_{st} of an economy, we characterize the equilibria of this benchmark case.

Proposition 1. Characterization of the Equilibria

- 1. Suppose the degree of child affinity is high. There is a unique equilibrium $\langle \lambda_{ut}, \lambda_{st} \rangle$ where all parents invest with certainty $\lambda_{ut} = \lambda_{st} = 1$.
- 2. Suppose the degree of child affinity is moderate. The unique equilibrium is such that
 - (a) when the state variable is higher than $\underline{b}_u(\delta)$, then the unskilled workers invest with a positive probability at which (3) binds and skilled workers invest with certainty,
 - (b) when the state variable is no higher than $\underline{b}_u(\delta)$ and no less than $\overline{b}_s(\delta)$, then the unskilled workers do not invest and the skilled workers invest with certainty,
 - (c) when the state variable is less than $\bar{b}_s(\delta)$, then the unskilled workers do not invest and the skilled workers invest with a positive probability at which (3) binds.
- 3. If and only if the degree of child affinity is low, there is a unique equilibrium such that
 - (a) when the state variable is no less than $b_s(\delta)$, then the unskilled workers do not invest and the skilled workers invest with certainty,
 - (b) when the state variable is less than $b_s(\delta)$ and higher than $\underline{b}_s(\delta)$, then no unskilled worker invests and the skilled workers invest with a probability at which (3) binds,
 - (c) when the state variable is no less than $\underline{b}_s(\delta)$ then no worker invests.



Figure 1: Characterization of the Equilibria in the Benchmark Case

We prove this in Appendix A.6 and depict the equilibria in Figure 1.

The intuition behind this proposition is, now, immediate. When the degree of child affinity is high, the parents care for their children so much that they invest at all relevant range of the state variable. When the degree of child affinity is moderate, the unskilled workers no longer invest with certainty and the probability of investment decreases with decrease in the state variable. If the state variable falls below $\underline{b}_u(\delta)$, then the unskilled workers do not invest at all. As discussed above, $\underline{b}_u(\delta)$ is negatively related to the parent's degree of child affinity. It becomes infinite when the degree of child affinity is low – an unskilled worker with low degree of child affinity never invests. The corresponding intuition for a skilled worker is similar. Only the thresholds are different as income of a skilled worker is higher which makes her utility cost of investment lower. Next, we analyze the dynamics and steady state of an economy. We say there is a *poverty* trap if there exists a positive mass of families that never become rich, which in our model corresponds to the adult working as a skilled worker. Alternatively, there is no poverty trap if the probability with which any family becomes rich at any particular period is positive.

Proposition 2. Dynamics and Steady States

- 1. When the degree of child affinity is not low, there is no poverty trap in the economy.
 - (a) When child affinity is high, the economy immediately reaches the steady state all parents invest, the mass of skilled worker is β and the income of a skilled worker is $A\beta^{-(1-\phi)}$. At any period, the probability with which a family becomes rich is β .
 - (b) When the degree of child affinity is moderate, if the mass of skilled worker is higher than $\beta \left(\underline{b}_u(\delta)/A\right)^{-\frac{1}{1-\phi}}$, then it decreases over time. At the steady state, the unskilled workers invest with a positive probability and the skilled workers invest with certainty. The mass of skilled worker is $\beta \left(\underline{b}_u(\delta)/A\right)^{-\frac{1}{1-\phi}}$ and the income of a skilled worker is $\beta^{-(1-\phi)}\underline{b}_u(\delta)$. At any period, the probability with which a family becomes rich is lower than β and it decreases with decrease in child affinity.
 - (c) The inequality at the steady state (weakly) increases with a decrease in child affinity – it remains constant when child affinity is high and strictly increases when it is (moderate).
- 2. When child affinity is low, if the state variable is
 - (a) higher than $\underline{b}_s(\delta)$, then the mass of skilled workers decreases over time and converges to zero, correspondingly their income converges to infinity.
 - (b) no higher than $\underline{b}_s(\delta)$, then the economy immediately enters into a steady state where all workers are unskilled and no parent invests. At the steady state, all families are in a poverty trap and there is no inequality.

We prove this in Appendix A.7.

When the degree of child affinity is high, all types of workers invest with certainty. Thus, the economy immediately reaches the steady state where all children are educated. At any period, a family becomes rich, i.e. the adult works as a skilled worker with probability β . So, there is no poverty trap. Since all parents invest with certainty at any $\delta \geq \overline{\delta}$, the inequality at the steady state – the difference between the income of a skilled worker and that of an unskilled worker – remains constant with the decrease in the degree of child affinity.

When the degree of child affinity is moderate, the unskilled workers no longer invest with certainty. When the initial mass of skilled worker is low such that the state variable is higher than $\underline{b}_u(\delta)$, then the expected benefit from investment is sufficiently large, and the unskilled workers invest with a positive probability. The economy immediately enters into the steady state. When the initial mass of skilled worker is not low such that the state variable is no higher than $\underline{b}_u(\delta)$, then the expected benefit from investment is small. Here no unskilled worker invests and the skilled workers invest with a positive probability. Hence, the mass of skilled worker increases over time. This happens

as long as the income becomes no less than $\underline{b}_u(\delta)$. The economy, then, enters into the steady state where all unskilled workers invest with a positive probability (and the skilled workers invest with probability 1). So, at any period, the probability with which a family becomes rich is positive. However, that probability is less than β because unskilled workers invest with probability less than 1 and at any period, the probability that the adult of a family works as an unskilled worker is positive. The steady state probability with which an unskilled worker invests decreases with decrease in the degree of child affinity. So, the probability with which a certain family becomes rich at a particular period decreases with child affinity.

The intuition behind the increase in inequality at the steady state with decrease in the degree of child affinity is quite obvious. Given the state variable, higher child affinity implies greater probability of investment which lowers future skilled incomes. Hence, within the range of moderate child affinity, the steady state inequality is decreasing in the child affinity parameter.

When child affinity is low, unskilled workers never invest. While the skilled workers invest but only a β fraction of their children are skilled workers in the next period, thus, the mass of skilled workers asymptotes to zero. In the steady state, everyone is unskilled, so there is no inequality.

Next, we address the main focus of this paper – the case where the parents are biased.

5 Behavioral Anomaly

The parents underestimate the probability of intergenerational mobility. While forming the beliefs about the probability of her child becoming a skilled worker, upon getting education, a parent looks through the lens of her education and job network. She discounts the possibility of her child becoming a worker of a different type than herself. We capture this through "degree of association". Adults of each network, in an economy, are characterized by their common degree of association with other network – the extent by which they relate to or feel connected with other networks. The degree of association of a Not Educated ⁶ worker with a Skilled worker is $\eta\theta$. The degree of association of an educated worker with a worker from the job network different from hers is θ . We assume that the degree of association decreases with social distance. Since education is necessary to become a skilled worker, Not Educated workers are the furthest from the Educated & Skilled workers – they neither belong to their education network nor to their job network. So, $\eta, \theta \in [0, 1]$.⁷

5.1 Bias via Education: Behavioral Trap

We start our analysis with the case where only not educated parents are biased, they do not believe that an educated child from their network would ever get a skilled job, i.e. $\eta = 0$.

⁶A word about notation: workers can be of three types – Not Educated & Unskilled, Educated & Unskilled and Educated & Skilled. Here, we need to denote unskilled workers – not educated and educated – differently, as they choose differently. For the brevity, we denote the former as Not Educated because without education it is not possible to get a skilled job and the latter as Educated & Unskilled. Similarly, as education is necessary for a skilled job, Educated & Skilled workers is denoted by Skilled workers.

⁷Observe, in the benchmark case, $\eta = \theta = 1$.

We may say lack of education imprisons not educated parents in a behavioral trap. The educated parents do not have any bias – they believe that the probability with which an educated child becomes a skilled worker is β , the true probability. In the next section, we relax this assumption and analyze the case where educated parents are also biased. A parent invests only when that provides her (weakly) higher utility than that from not investing. The immediate implication of η being zero is

Observation 3. In presence of a behavioral trap due to lack of education, Not Educated workers never invest.

The educated workers take this into account and invest. Let the probability with which an Educated & Unskilled worker invests at period t be ρ_{ut}^{8} and that for a Skilled worker be ρ_{st} . At period t, a worker of type j invests in child's education with probability ρ_{jt} if and only if

$$\delta\left[\frac{\left[\beta^{\phi}A[\rho_{ut}(1-\beta)N_{et}+\rho_{st}\beta N_{et}]^{-(1-\phi)}+1-\beta\right]^{\sigma}}{\sigma}-\frac{1}{\sigma}\right] \ge \frac{m_{jt}^{\sigma}}{\sigma}-\frac{(m_{jt}-\bar{s})^{\sigma}}{\sigma}.$$
 (4)

recall N_{et} is the mass of educated workers, $(1-\beta)N_{et}$ is that of Educated & Unskilled workers and βN_{et} is that of Skilled workers. The inequality binds for j^{th} type when $\rho_{jt} \in (0, 1)$.

An equilibrium is denoted by $\langle \rho_{ut}, \rho_{st} \rangle$ which satisfies the features described in Subsection 3.2. Like in the benchmark case, we have the following

Observation 4. Consider any equilibrium $\langle \rho_{ut}, \rho_{st} \rangle$

- (i) if Educated & Unskilled workers invest with a positive probability ($\rho_{ut} > 0$), then all Skilled workers invest with certainty ($\rho_{st} = 1$),
- (ii) income of a Skilled worker is the state variable.

The proof is very similar to that of Lemma 1, so we skip it here.

Due to the existence of the behavioral trap, there does not exist any degree of child affinity where all parents invest. We define the following new threshold of the state variable.

Definition 3. Let $\langle \rho_{ut}, \rho_{st} \rangle$ be an equilibrium at the state variable m_{st} . For a given degree of child affinity, $\bar{b}_u(\delta)$ is the minimum value of the state variable (m_{st}) at period t, at which Educated & Unskilled workers invest with certainty, i.e. $\rho_{ut} = 1$ if and only if $m_{st} \geq \bar{b}_u(\delta)$.

We provide the formal expression of this threshold of the state variable in Appendix B.1. This new threshold along with the thresholds defined in Definition 2 are used for further analyses. Those thresholds are relevant here, as at those thresholds, unskilled workers do not invest – they invest only when the state variable is higher than $\underline{b}_u(\delta)$ which is the highest among them. So the difference due to the non-investment of Not Educated workers under behavioral trap does not affect those thresholds and hence, are valid here. The observation below documents some features of this new threshold and its ranking among other thresholds.

Observation 5. (i) $\bar{b}_u(\delta)$ is decreasing in δ .

⁸Here, unlike the benchmark case, subscript u denotes Educated & Unskilled. Not Educated workers never invest, so this is for the brevity of notation.

(ii) $\bar{b}_u(\bar{\delta}) = A\beta^{-(1-\phi)}$, when the degree of child affinity is moderate $\bar{b}_u(\delta) = \beta^{-(1-\phi)}\underline{b}_u(\delta)$ and when the degree of child affinity is low, then $\bar{b}_u(\delta) = \infty$.

We prove this observation in Appendix B.2.

Given the parameters $\delta, \sigma, \bar{s}, \beta, \eta$ and state variable m_{st} of an economy, we characterize the equilibria in the following proposition.

Proposition 3. Characterization of the Equilibria

- 1. The Not Educated workers never invest.
- 2. Suppose the degree of child affinity is high, then at the unique equilibrium all educated workers invest with certainty.
- 3. Suppose the degree of child affinity is moderate. The unique equilibrium is such that when the state variable is
 - (a) no less than $b_u(\delta)$, then all educated workers invest with probability 1,
 - (b) lower than $\bar{b}_u(\delta)$ and higher than $\underline{b}_u(\delta)$, then Educated & Unskilled workers invest with a probability at which (4) binds and the Skilled workers invest with certainty,
 - (c) no higher than $\underline{b}_u(\delta)$ and no less than $\underline{b}_u(\delta)$, then Educated & Unskilled workers do not invest and the Skilled workers invest with probability 1,
 - (d) no less than $\underline{b}_u(\delta)$, then Educated & Unskilled workers do not invest and the Skilled workers invest with a probability at which (4) binds.
- 4. Suppose child affinity is low, unique equilibrium is such that when the state variable is
 - (a) no less than $\bar{b}_s(\delta)$, then unskilled workers do not invest and skilled workers invest with certainty,
 - (b) less than $\bar{b}_s(\delta)$ and higher than $\underline{b}_s(\delta)$, then unskilled workers do not invest and skilled workers invest with a positive probability at which (4) binds,
 - (c) no less than $\underline{b}_s(\delta)$ then no worker invests.



Figure 2: Characterization of the Equilibria with a Behavioral Trap

We prove this in Appendix B.3 and depict the equilibria in Figure 4.

A word about why we are getting $\rho_{ut} = 1$ when the degree of child affinity is moderate, though in the benchmark case, λ_{ut} is always less than 1. The reason is very simple. First note that $\bar{b}_u(\delta) > \underline{b}_u(\delta)$, so the state variable for which $\gamma_{ut} = 1$, $\lambda_{ut} > 0$. Hence, when the mass of Not Educated workers is positive, they were investing in the benchmark case and not under the behavioral trap. This, in general, makes $\gamma_{ut} > \lambda_{ut}$ whenever $\lambda_{ut} \in (0, 1)$ and the mass of Not Educated workers is positive: From Equations (3), (4), Observation 2 and 5 we have

$$\lambda_{ut}L_{ut} + L_{st} = \gamma_{ut}(1-\beta)N_{et} + \beta N_{et} \qquad \Rightarrow \qquad \lambda_{ut}\underbrace{[1-N_{et}]}_{\text{Not Educated}} = (\gamma_{ut} - \lambda_{ut})\underbrace{(1-\beta)N_{et}}_{\text{Educated & Unskilled}}$$

Hence, when there is a behavioral trap and $\gamma_{ut} \in (0, 1)$, the non investment of the Not Educated workers is exactly compensated by the over investment of Educated & Unskilled workers, such that in both the cases the mass of skilled workers in the next period and hence, the benefits from investment remain the same as the cost of investment of an unskilled worker is the same in both the cases. Now, it is clear from the definition that at $\bar{b}_u(\delta)$ we must have

$$\lambda_u^*(1 - N_{et}) = (1 - \lambda_u^*)(1 - \beta)N_{et}$$

where recall λ_u^* is the probability with which unskilled workers invest at the steady state in the benchmark case such that the mass of educated workers remain constant over time. Here, also observe that happens when $\gamma_{ut} = 1$. This implies at each δ , there is a unique value of N_{et} corresponding to $\bar{b}_u(\delta)$. If N_{et} is higher than that value then Educated & Unskilled workers would not invest with probability 1, and if that is lower than that value, then the benefit from investment of an unskilled worker would be strictly higher than her cost of investment. Yet, due to the behavioral trap, Not Educated workers would not invest, and that gives rise to multiple steady states. An economy converges to one of those depending on the mass of Not Educated workers. The steady states can be ranked according to inequality – the difference between the income of a skilled worker and that of an unskilled worker. When the degree of child affinity is high, the steady state where $m_s^* = \bar{b}_u(\bar{\delta})$, recall, which is equal to $A\beta^{-(1-\phi)}$, is called the 'least unequal steady state'. When child affinity is moderate, the steady state where $m_s^* = \bar{b}_u(\delta)$ we call that the 'least unequal steady state'.

Proposition 4. Dynamics and Steady States

- 1. There is almost always a poverty trap in an economy there is no poverty trap in an economy only when the economy starts with all educated parents, and $\delta \geq \overline{\delta}$.
- 2. When the degree of child affinity is not low there are multiple steady states.
 - (a) When the degree of child affinity is high any $m_s^* \ge 1$ is a steady state. The economy immediately reaches a steady state, the mass of each type of workers remains constant at the level with which the economy starts.
 - (b) When child affinity is moderate, if $m_{st} \ge b_u(\delta)$, then all educated workers invest with certainty. The mass of educated individuals and that of skilled workers remain

constant over time. The income of a skilled worker remains constant at $m_{st} \equiv m_s^*$.

- (c) When the degree of child affinity is moderate, if $m_{st} < \bar{b}_u(\delta)$, at least one type of workers do not invest with certainty. The mass of educated individuals and the mass of skilled workers decrease over time. The income of a skilled worker increases over time and converges to some $m_{st} \equiv m_s^* \geq \bar{b}_u(\delta)$.
- (d) The multiple steady states can be ranked on the basis of inequality higher the m_s^* higher is the inequality.
- (e) The inequality at the least unequal steady state (weakly) increases with decrease in the degree of child affinity – it remains constant when the degree of child affinity is high and it strictly increases when the degree of child affinity is moderate.
- 3. When the degree of child affinity is low.
 - (a) If $m_{st} > \underline{b}_s(\delta)$, then the mass of skilled workers decreases over time and converges to zero, and correspondingly the income of a skilled worker converges to infinity.
 - (b) If $m_{st} \leq \underline{b}_s(\delta)$, the economy immediately reaches a steady state where all workers are unskilled.
 - (c) There is no inequality at the steady state.

We prove this in Appendix B.4.

Interestingly, observe with a behavioral trap, as Not Educated workers never invest, it is not possible to have mixed strategies being played in any steady state. As that would decrease the mass of educated workers, hence the mass of Skilled workers in the next period, and that cannot be a steady state. For a similar reason, with a behavioral trap, we have multiple steady states – even if there are a few educated families in an economy which makes the mass of skilled workers small and their income large, due to their behavioral imprisonment, Not Educated workers do not invest and the economy stays there forever.

We discuss more about the implications of behavioral trap in Section 6. Next, we discuss the case where the educated parents are also biased.

5.2 Bias via Education and Job Network: Behavioral Trap & Behavioral Bias

All parents, here, are biased. Recall, the degree of association via job network is captured by θ , here $\theta \in (0, 1)$. An Educated & Unskilled worker believes the probability with which an educated child from her community becomes a skilled worker is $\theta\beta$. A Skilled worker believes that an educated child from her own network will become an unskilled worker with probability $\theta(1 - \beta)$. So, she believes the probability that such a child will become a skilled worker is $1 - \theta(1 - \beta)$. Like before a Not Educated worker believes that a child from her own network, upon getting education, will become a skilled worker with probability $\eta\theta\beta$, and we continue to assume η to be zero. Hence, the Not Educated workers do not invest here also. As $\theta < 1$, Educated & Unskilled workers are under confident and Skilled workers are over confident.⁹ This captures the behavioral anomaly – parents' experiences color their perceptions about their children. We assume, that while calculating the probability with which an educated child from a different network becomes a skilled worker, an individual can see clearly. An unskilled (skilled) worker, correctly, believes that an educated child of a skilled (unskilled) worker becomes a skilled worker with probability β .

Since a parent's belief about the probability of success – becoming a skilled worker upon getting education – of a child from her own network is type dependent, the 'conjectured' mass of skilled workers and their income in the next period would also be type dependent. To characterize the investment decision of each types of parents, we now discuss the conjectured expected benefit from investment. Suppose, at period t, a worker of type j, where $j \in$ $\{u, s\}$,¹⁰ invests with probability γ_{jt} . Then an Educated & Unskilled worker conjectures that the mass of skilled workers and their income at period t + 1 would be

$$L_{st+1}^{u} = \theta \beta \cdot \gamma_{ut} (1-\beta) N_{et} + \beta \cdot \gamma_{st} \beta N_{et}, \quad \text{and } \omega_{st+1}^{u} = A [\theta \beta \cdot \gamma_{ut} (1-\beta) N_{et}]^{-1-\phi}.$$

where, recall, N_{et} denotes the mass of educated individuals at period t, hence, $(1 - \beta)N_{et}$ is the mass of Educated & Unskilled workers who invest with probability γ_{ut} and βN_{et} is the mass of skilled workers who invest with probability γ_{st} .

Thus, the conjectured benefit from investment of an Educated & Unskilled worker is

$$\theta\beta \cdot \left[A[\theta\beta \cdot \gamma_{ut}(1-\beta)N_{et}]^{-1-\phi}\right] + 1 - \theta\beta = \theta\beta[\theta\gamma_{ut}(1-\beta) + \beta\gamma_{st}]^{-(1-\phi)}m_{st} + 1 - \theta\beta.$$

Similarly, a Skilled worker conjectures that the mass of skilled workers and their income at period t + 1 would be

$$L_{st+1}^s = \beta \cdot \gamma_{ut}(1-\beta)N_{et} + [1-\theta(1-\beta)] \cdot \gamma_{st}\beta N_{et} \quad \text{and} \ \omega_{st+1}^s = AL_{st+1}^{s^{-(1-\phi)}}.$$

And, the conjectured benefit from investment of a Skilled worker is

$$[1 - \theta(1 - \beta)] \cdot [\gamma_{ut}(1 - \beta) + [1 - \theta(1 - \beta)]\gamma_{st}]^{-(1 - \phi)}m_{st} + \theta(1 - \beta).$$

Observe, here the "conjectured" benefits of the two types of workers cannot be ranked. This is because the under (over) confident Educated & Unskilled (Skilled) workers under (over) estimates the mass of future skilled workers and hence, over (under) estimates their income.

At any period t, an Educated & Unskilled worker invests in her child's education with probability γ_{ut} if and only if

$$\frac{(1-\bar{s})^{\sigma}}{\sigma} + \delta \frac{\left[\theta\beta[\theta\gamma_{ut}(1-\beta)+\beta\gamma_{st}]^{-(1-\phi)}m_{st}+1-\theta\beta\right]^{\sigma}}{\sigma} \ge \frac{1}{\sigma} + \frac{\delta}{\sigma}$$

$$\Rightarrow \quad \delta \left[\frac{\left[\theta\beta[\theta\gamma_{ut}(1-\beta)+\beta\gamma_{st}]^{-(1-\phi)}m_{st}+1-\theta\beta\right]^{\sigma}}{\sigma} - \frac{1}{\sigma}\right] \ge \frac{1}{\sigma} - \frac{(1-\bar{s})^{\sigma}}{\sigma}.$$
(5)

 ${}^{9}\theta\beta < \beta \leq 1 - \theta(1 - \beta).$

¹⁰Here also, we use subscript u for Educated & Unskilled workers and subscript s for the Skilled workers.

The L.H.S. is the *conjectured* net benefit and the R.H.S. is the net utility cost from investment. Similarly, at any period t, a Skilled worker invests in her child's education with probability γ_{st} if and only if

$$\delta \left[\frac{\left[\left[1 - \theta(1-\beta) \right] \cdot \left[\gamma_{ut}(1-\beta) + \left[1 - \theta(1-\beta) \right] \gamma_{st} \right]^{-(1-\phi)} m_{st} + \theta(1-\beta) \right]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right]$$

$$\geq \frac{m_{st}}{\sigma} - \frac{(m_{st} - \bar{s})^{\sigma}}{\sigma}. \tag{6}$$

We observed that the utility cost of investment is lower for a Skilled worker. But, as their *conjectured* net benefit from investment cannot be ranked, we find that Part (a) of Lemma 1 is not true anymore – an Educated & Unskilled worker may invest with a positive probability, even when Skilled workers do not invest with certainty. However, we find

Lemma 3. At any equilibrium, if Educated & Unskilled workers invest, then Skilled workers invest with positive probability: suppose $\langle \gamma_{ut}, \gamma_{st} \rangle$ is an equilibrium, and $\gamma_{ut} > 0$ then $\gamma_{st} > 0$.

We prove this in Appendix C.1.

Intuitively, we observe that when no Skilled workers invest and Educated & Unskilled workers invest then the *conjectured* benefit of a Skilled worker is higher than that of an Educated & Unskilled worker. And, we have already observed that the utility cost of a Skilled worker is lower due to the concavity of utility function. Hence, the lemma.

We, now, define an additional threshold of degree of child affinity for further analyses.

Definition 4. The degree of child affinity is huge when $\delta \geq \delta_a$, where

$$\delta_a \equiv \frac{(1-\bar{s})^{\sigma} - 1}{1 - [\theta\beta(\theta(1-\beta) + \beta)^{-(1-\phi)} + 1 - \theta\beta]^{\sigma}}.$$

 δ_a along with $\underline{\delta}$ as in Definition 1 characterize the equilibria, here. The ranking is follows.

Observation 6. $0 < \underline{\delta} < \delta_a$.

Appendix C.2 proves this. The degree of child affinity is *moderately high* when $\underline{\delta} \leq \delta < \delta_a$ and recall *low* when $\delta < \underline{\delta}$.

The next observation follows directly from the optimal investment decisions of Educated & Unskilled workers captured in (5) and that of Skilled workers stated in (6).

Observation 7. Suppose at any m_{st} , when workers of type k invest with probability γ_{kt} , the workers of type j optimally invest with probability γ_{jt} , where $k, j = \{u, s\}$ and $k \neq j$. Then at any $\tilde{m}_{st} > m_{st}$, when workers of type k invest with probability no higher than γ_{kt} , the workers of type j optimally invest with probability no less than γ_{jt} .

Now, we introduce various thresholds of the state variable m_{st} . The first threshold, like in the benchmark case, characterizes an equilibrium. The rest of the thresholds address optimal decisions – second, third and the fourth are thresholds dealing with the optimal decisions of the Skilled workers *if* they believe that the Educated & Unskilled workers choose the mentioned γ_{ut} . Note that there may not exist any equilibrium at such a threshold where the mentioned $\langle \gamma_{ut}, \gamma_{st} \rangle$ is an equilibrium or there may exist many other equilibria at that threshold. The last three thresholds are similarly for the optimal decisions of the Educated & Unskilled workers *if* they believe that the Skilled workers choose the mentioned γ_{st} .

Definition 5. For a given degree of child affinity,

(i) suppose $\langle \gamma_{ut}, \gamma_{st} \rangle$ is an equilibrium. $\underline{\mathbf{a}}_{s}(\delta)$ is the maximum value of the state variable at which the Skilled workers do not invest, i.e.

 $\gamma_{st} = 0$ if and only if $m_{st} \leq \underline{a}_s(\delta)$.

(ii) suppose the Educated & Unskilled workers do not invest, then $\mathbf{a}_{6}(\delta)$ is the minimum value of the state variable at which Skilled workers invest with certainty, i.e.

suppose $\gamma_{ut} = 0$, then, $\gamma_{st} = 1$ if and only if $m_{st} \ge a_6(\delta)$,

(iii) suppose the Educated & Unskilled workers invest with probability 1, then $\mathbf{a}_4(\delta)$ is the maximum value of the state variable at which Skilled workers do not invest, i.e.

suppose
$$\gamma_{ut} = 1$$
, then, $\gamma_{st} > 0$ if and only if $m_{st} > a_4(\delta)$,

(iv) suppose the Educated & Unskilled workers invest with probability 1, then $\mathbf{a}_2(\delta)$ is the minimum value of the state variable at which Skilled workers invest with certainty, i.e.

suppose $\gamma_{ut} = 1$, then, $\gamma_{st} = 1$ if and only if $m_{st} \ge a_2(\delta)$,

(v) suppose the Skilled workers invest with probability 1, then $\mathbf{a}_{\mathbf{5}}(\delta)$ is the maximum value of the state variable at which Educated & Unskilled workers do not invest, i.e.

Suppose $\gamma_{st} = 1$, then, $\gamma_{ut} > 0$ if and only if $m_{st} > a_5(\delta)$,

(vi) suppose the Skilled workers do not invest, then $\mathbf{a}_{\mathbf{3}}(\delta)$ is the minimum value of the state variable at which Educated & Unskilled workers invest with certainty, i.e.

Suppose
$$\gamma_{st} = 0$$
, then, $\gamma_{ut} = 1$ if and only if $m_{st} \ge a_3(\delta)$,

(vii) suppose the Skilled workers invest with probability 1, then $\mathbf{a}_2(\delta)$ is the minimum value of the state variable at which Educated & Unskilled workers invest with certainty, i.e.

Suppose
$$\gamma_{st} = 1$$
, then, $\gamma_{ut} = 1$ if and only if $m_{st} \ge a_1(\delta)$,

The formal expressions for these thresholds are given in Appendix C.3. For further analyses, in the following lemma, we collect important features of the thresholds.

Lemma 4. Properties of the thresholds of the state variable

- i. All thresholds of the state variable are decreasing in δ .
- *ii.* The thresholds related to the Skilled workers' investment decisions are such that:
 - (a) $\forall \delta > 0$, we have $1 < a_2(\delta)$.
 - (b) $\forall \delta > 0$, we have $a_{\mathfrak{s}}(\delta) < a_6(\delta) < a_2(\delta)$ and $a_{\mathfrak{s}}(\delta) < a_4(\delta) < a_2(\delta)$.
 - (c) $\forall \delta > 0, a_4(\delta) > a_6(\delta)$ if and only if $\theta(1-\beta) > \beta$.
- *iii.* The thresholds related to the Educated & Unskilled workers' decisions are such that:
 - (a) $a_1(\delta)$, $a_3(\delta)$ and $a_5(\delta)$ are finite if and only if $\delta > \underline{\delta}$.
 - (b) $1 < a_1(\delta)$ if and only if $\delta < \delta_a$.
 - (c) $\forall \delta > \delta$, we have $a_5(\delta) < a_1(\delta)$ and $a_3(\delta) < a_1(\delta)$.
 - (d) $\forall \delta > \delta$, $a_3(\delta) > a_5(\delta)$ if and only if $\theta(1-\beta) > \beta$.
- iv. $\forall \delta \leq \delta_a$, we have $a_4(\delta) \leq a_1(\delta)$ and $\forall \delta > \delta_a$, we have $a_4(\delta) < 1$.
- v. Cut-off properties relative to the benchmark case are:

(a)
$$\underline{b}_s(\delta) = \underline{a}_s(\delta)$$
.

(b)
$$\underline{b}_u(\delta) < a_5(\delta)$$
.

We show the proofs in Appendix C.4.

Next, we discuss the intuition of this lemma. But before that a word about the parametric condition $\theta(1-\beta) > \beta$. This and the converse of it are important in optimal decisions of both types of workers, and hence in the characterization of the equilibria. Recall, θ captures the degree of association via job network. N_{et} denotes the mass of educated workers, and due to the presence of behavioral trap, it also represents the mass of potential parents who may invest. Among them $(1 - \beta)$ is the proportion of Educated & Unskilled workers and β is that of the Skilled workers. Now, $\theta(1-\beta) > \beta$ implies, first, the degree of association is higher than the proportion of skilled and unskilled workers among the educated individuals. Second, when $\gamma_{ut} = \gamma_{st}$

$$\theta\beta(1-\beta)N_{et}$$

Educated & Unskilled workers' conjectured mass of skilled workers in the next period coming from their community

Educated & Unskilled workers' conjectured mass of skilled workers in the next period coming from the community of Skilled workers

Skilled workers' conjectured mass of skilled workers in the next period coming from their community

and $\beta [1 - \theta (1 - \beta)] N_{et} < \beta (1 - \beta) N_{et}$

Skilled workers' conjectured mass of skilled workers in the next period coming from the community of Educated & Unskilled workers

So, if both types of workers invest with the same probability, then workers of both types conjecture that the contribution of the Educated & Unskilled workers in the mass of skilled workers of the next period is higher than that contribution of the Skilled workers.

Now, we provide the intuition of some of the properties of the thresholds depicted in Lemma 4. Observe, on the one hand, the benefit from investment of any type of worker is decreasing in the conjectured mass of skilled workers and is increasing with the state variable. On the other hand, the cost of investment is non-increasing in the state variable – it is decreasing for the Skilled workers and constant for the Educated & Unskilled workers. Hence, the ranking of the thresholds depend on the conjectured mass of skilled workers at the primitives of the definitions. Higher is that mass higher is the threshold. For example, at the primitives of $\underline{a}_s(\delta)$ the conjectured mass of Skilled worker is zero whereas that at $a_6(\delta)$ is positive. Hence, $a_6(\delta)$ must be higher than $\underline{a}_s(\delta)$ – the Skilled workers are willing to invest at the primitive of $\underline{a}_s(\delta)$ when the state variable is low as the benefit is infinity whereas the state variable must be higher to make them invest at the primitive of $a_6(\delta)$. The other ranking between $a_4(\delta)$ and $a_6(\delta)$ follows from the discussion on $\theta(1 - \beta) > \beta$ and this logic. The same goes for the ranking between $a_3(\delta)$ and $a_5(\delta)$.

Second, observe that the state variable, by assumptions, cannot be less than 1. So, if we find that any threshold of the state variable is less than 1, then when the primitive of the definition is satisfied, then the optimal strategy described in the definition would always be true at all permissible range of the state variable. For example, we show that $a_1(\delta) < 1$ when $\delta > \delta_a$. This implies when the degree of child affinity is huge and all Skilled workers invest with certainty, irrespective of the value of state variable, the Educated & Unskilled workers would optimally invest with certainty. Finally, the intuition behind the ranking between $a_4(\delta)$ and $a_1(\delta)$ follows directly from Lemma 3.

Now, we provide boundary conditions on equilibrium strategies. The first two conditions provide lower and upper bounds, respectively, on the equilibrium strategy of the Skilled workers and the last two provide the same of the Educated & Unskilled workers.

Lemma 5. Boundary Conditions on Equilibrium Probabilities of Investments

(i) Condition $\underline{\Gamma}_{s}$: Suppose $m_{st} \ge a_{4}(\delta)$. Then in any equilibrium $\gamma_{st}(m_{st})$ is bounded below by $\gamma_{s}(m_{st})$ which is given by

$$\delta \left[\frac{\left[\left[1 - \theta(1-\beta) \right] \cdot \left[(1-\beta) + \left[1 - \theta(1-\beta) \right] \underline{\gamma}_s(m_{st}) \right]^{-(1-\phi)} m_{st} + \theta(1-\beta) \right]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right]$$
$$\geq \frac{m_{st}}{\sigma} - \frac{(m_{st} - \bar{s})^{\sigma}}{\sigma}$$

the above inequality binds when $m_{st} \in [a_4(\delta), a_2(\delta)]$. We have $\underline{\gamma}_s(m_{st}) = 0$ at $m_{st} = a_4(\delta)$, $\underline{\gamma}_s(m_{st}) \in (0, 1) \quad \forall m_{st} \in (a_4(\delta), a_2(\delta)), \quad and \underline{\gamma}_s(m_{st}) = 1 \text{ at } m_{st} \ge a_2(\delta). \text{ Also,}$ $\underline{\gamma}_s(m_{st}) \text{ is nondecreasing in } m_{st} \ge a_4(\delta) \text{ and is strictly increasing when } m_{st} \in [a_4(\delta), a_2(\delta)).$ (ii) Condition $\bar{\Gamma}_s$: Suppose $m_{st} \leq a_6(\delta)$. Then in any equilibrium γ_{st} is bounded above by $\bar{\gamma}_s(m_{st})$ which is given by

$$\delta \left[\frac{\left[\left[1 - \theta(1-\beta) \right]^{\phi} \left[\underline{\gamma}_s(m_{st}) \right]^{-(1-\phi)} m_{st} + \theta(1-\beta) \right]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right] \le \frac{m_{st}}{\sigma} - \frac{(m_{st} - \bar{s})^{\sigma}}{\sigma}$$

where the above inequality binds when $m_{st} \in [\underline{a}_s(\delta), a_6(\delta))]$. We have $\bar{\gamma}_s(m_{st}) = 0$ at $m_{st} \leq \underline{a}_s(\delta), \ \bar{\gamma}_s(m_{st}) \in (0,1) \ \forall m_{st} \in (\underline{a}_s(\delta), a_6(\delta)), \ and \ \bar{\gamma}_s(m_{st}) = 1$ at $m_{st} = a_6(\delta)$. Also, $\bar{\gamma}_s(m_{st})$ is strictly increasing $\forall m_{st} \in [\underline{a}_s(\delta), a_6(\delta))$.



Figure 3: Condition $\underline{\Gamma}_s$ and $\overline{\Gamma}_s$

(iii) Condition $\underline{\Gamma}_{\mathbf{u}}$: Suppose $m_{st} \ge a_5(\delta)$. Then in any equilibrium γ_{ut} is bounded below by $\underline{\gamma}_u(m_{st})$ which is given by

$$\delta\left[\frac{\left[\theta\beta[\theta\underline{\gamma}_{u}(m_{st})(1-\beta)+\beta]^{-(1-\phi)}m_{st}+1-\theta\beta\right]^{\sigma}}{\sigma}-\frac{1}{\sigma}\right] \geq \frac{1}{\sigma}-\frac{(1-\bar{s})^{\sigma}}{\sigma}$$

the above inequality binds when $m_{st} \in [a_5(\delta), a_1(\delta)]$. Note, $\underline{\gamma}_u(m_{st}) = 0$ at $m_{st} = a_5(\delta)$, $\underline{\gamma}_u(m_{st}) \in (0, 1) \ \forall m_{st} \in (a_5(\delta), a_1(\delta))$, and $\underline{\gamma}_u(m_{st}) = 1$ at $m_{st} \ge a_1(\delta)$. Also, $\underline{\gamma}_u(m_{st})$ is nondecreasing in $m_{st} \ge a_5(\delta)$. It is strictly increasing when $m_{st} \in [a_5(\delta), a_1(\delta))$.

(iv) Condition $\bar{\Gamma}_{\mathbf{u}}$: Suppose $m_{st} \leq a_3(\delta)$. Then in any equilibrium γ_{ut} is bounded above by

 $\bar{\gamma}_u(m_{st})$ which is given by

$$\delta\left[\frac{\left[\theta\beta[\theta(1-\beta)\bar{\gamma}_u(m_{st})]^{-(1-\phi)}m_{st}+1-\theta\beta\right]^{\sigma}}{\sigma}-\frac{1}{\sigma}\right] \leq \frac{1}{\sigma}-\frac{(1-\bar{s})^{\sigma}}{\sigma}$$

We have $\bar{\gamma}_u(m_{st}) = 1$ at $m_{st} = a_3(\delta)$,

 $\bar{\gamma}_u(m_{st}) < 1 \ \forall m_{st} < a_3(\delta) \ and \ \bar{\gamma}_u(m_{st}) \ is \ non-decreasing \ in \ m_{st}.$

Note that, due to Lemma 3, when $m_{st} \leq a_3(\delta)$, at any equilibrium $\langle \gamma_{ut}, \gamma_{st} \rangle$, γ_{ut} will be strictly lower than $\bar{\gamma}_u(m_{st})$.

We prove this in Appendix C.5. The figure depicting Condition $\underline{\Gamma}_u$ and Condition $\overline{\Gamma}_u$ is very similar to Figure 3 which depicts Condition $\underline{\Gamma}_s$ and Condition $\overline{\Gamma}_s$, so we skip it in the main text, it can be found in Supplementary Appendix.

A word about how we get these boundary conditions. To understand take $\underline{\gamma}_s(m_{st})$ for example. First observe the way the $\underline{\gamma}_s(m_{st})$ has been defined in Condition $\overline{\Gamma}_s - if \gamma_{ut} = 1$ then at $\underline{\gamma}_s(m_{st})$, (6) binds $\forall m_{st} \in [a_4(\delta), a_2(\delta)]$ and the benefit (the L.H.S. of (6)) is strictly higher than the cost (R.H.S.) $\forall m_{st} > a_2(\delta)$. Second, the maximum value γ_{ut} can take is 1, so from (6), we can see that γ_{st} cannot be lower than $\underline{\gamma}_s(m_{st})$ for any $m_{st} \ge a_4(\delta)$, it is exactly equal to $\underline{\gamma}_s(m_{st})$ when $\gamma_{ut} = 1$. Hence, $\underline{\gamma}_s(m_{st})$ is the lower bound on $\gamma_{st} \forall m_{st} \ge a_4(\delta)$. Finally, we can see from the definition of $a_4(\delta)$, $a_2(\delta)$ and (6) that $\underline{\gamma}_s(a_4(\delta)) = 0$, $\underline{\gamma}_s(a_2(\delta)) = 1$ and $\underline{\gamma}_s(m_{st})$ is strictly increasing in $m_{st} \forall m_{st} \in [a_4(\delta), a_2(\delta)]$. Similar intuition follows for Conditions $\overline{\Gamma}_s$, $\underline{\Gamma}_u$ and $\overline{\Gamma}_u$.

Next, given the parameters $\delta, \sigma, \bar{s}, \beta, \eta, \theta$, and state variable m_{st} of an economy, we characterize the equilibria of this benchmark case.

Proposition 5. Characterization of the Equilibria

- 1. The Not Educated workers never invest.
- 2. If and only if the degree of child affinity is huge, i.e. $\delta > \delta_a$, at any $m_{st} \ge 1$, there exists a unique equilibrium $\langle \gamma_{ut}, \gamma_{st} \rangle$ and is given by

 $\gamma_{ut} = 1$ and $\gamma_{st} = \underline{\gamma}_s(m_{st})$; where $\underline{\gamma}_s(m_{st})$ is as defined in Condition $\underline{\Gamma}_s$ in Lemma 5.

- 3. If and only if the degree of child affinity is moderately high, i.e. $\delta \in (\underline{\delta}, \delta_a]$
 - (i) at any $m_{st} \geq \min\{a_1, a_2\}$, there exists a unique equilibrium $\langle \gamma_{ut}, \gamma_{st} \rangle$ given by

$$\begin{cases} If \ a_1(\delta) \leq a_2(\delta) & \gamma_{ut} = 1, \ and \ \gamma_{st} = \underline{\gamma}_s(m_{st}) & \forall m_{st} \geq a_1(\delta) \\ & where \ \underline{\gamma}_s(m_{st}) \ is \ as \ defined \ in \ Condition \ \overline{\Gamma}_s \ in \ Lemma \ 5 \\ \\ If \ a_1(\delta) > a_2(\delta) & \gamma_{ut} = 0, \ and \ \gamma_{st} = 1 & \forall m_{st} \in [a_2(\delta), a_5(\delta)), \\ & \gamma_{ut} = \underline{\gamma}_u(m_{st}), \ and \ \gamma_{st} = 1 & \forall m_{st} \geq a_5(\delta) \\ & where \ \underline{\gamma}_u(m_{st}) \ is \ as \ defined \ in \ Condition \ \overline{\Gamma}_u \ in \ Lemma \ 5. \end{cases}$$

- (ii) at any $m_{st} \in [1, \min\{a_1(\delta), a_2(\delta)\})$, at any equilibrium $\langle \gamma_{ut}, \gamma_{st} \rangle$, at least one type of educated parents invest with a positive probability. In any equilibrium $\langle \gamma_{ut}, \gamma_{st} \rangle$, Conditions $\underline{\Gamma}_s$, $\overline{\Gamma}_s$, $\underline{\Gamma}_u$ and $\overline{\Gamma}_u$, defined in Lemma 5, must be satisfied.
 - (a) Suppose $\beta < \theta(1-\beta)$. There is a unique equilibrium at any $m_{st} \ge 1$. If $a_1(\delta) < a_2(\delta)$, then $\gamma_{st} < \gamma_{ut}$.
 - (b) Suppose $\beta > \theta(1-\beta)$. If there are multiple equilibria at any $m_{st} \ge 1$, then at most in one such equilibrium both types of workers would play mixed strategies.
- 4. If and only if the degree of child affinity is low, i.e. $\delta < \underline{\delta}$, at any $m_{st} \ge 1$, there exists a unique equilibrium $\langle \gamma_{ut}, \gamma_{st} \rangle$ and is given by

$$\begin{cases} \gamma_{ut} = 0, \text{ and } \gamma_{st} = \bar{\gamma}_s(m_{st}) & \forall m_{st} \in [1, a_6(\delta)] \\ \text{where } \bar{\gamma}_s(m_{st}) \text{ is as defined in Condition } \bar{\Gamma}_s \text{ in Lemma 5} \\ \gamma_{ut} = 0, \text{ and } \gamma_{st} = 1 & \forall m_{st} > a_6(\delta). \end{cases}$$



Figure 4: Characterization of the Equilibria with a Behavioral Trap

We prove this in Appendix C.6.

Let us depict some numerical examples where $\theta(1-\beta) < \beta$. As noted, for such parameter range multiple paths may exist or not exist. For $\theta = 0.4$ and $\beta = 0.7$, we show that unique equilibrium exists for all m_{st} in Figure 5 while multiple equilibria exists for some m_{st} in Figure 6. Existence of multiple equilibria may lead to behavioral cycles in investment. If adults in period t invest independent of investment decisions in the previous period, they may choose different equilibrium compared to their predecessors. These fluctuations in educational investment is not due to any fundamental changes in the economy, but only due to existence of multiple equilibria.



Figure 5: Example to depict unique equilibrium for all $m_{st} > 1$ and $\theta(1 - \beta) < \beta$.



Figure 6: Example to depict multiple equilibria for some $m_{st} > 1$ and $\theta(1 - \beta) < \beta$.

We, now, analyze the dynamics and steady states. Here also, as the Not Educated workers never invest, we have multiple steady states and they can be ranked in terms of inequality. The steady state where $m_s^* = \max\{a_1(\delta), a_2(\delta)\}$, we call that the 'least unequal steady state'.

Proposition 6. Dynamics and Steady States

- 1. There is almost always a poverty trap in an economy there is none only when the economy starts with all educated parents, $\max\{a_1(\delta), a_2(\delta)\} \leq A\beta^{-(1-\phi)}$, and $\delta > \underline{\delta}$.
- 2. When the degree of child affinity is not low there are multiple steady states any $m_s \ge \max\{a_1(\delta), a_2(\delta)\}$ is a steady state.
 - (a) If $m_{st} \ge \max\{a_1(\delta), a_2(\delta)\}$, then all educated workers invest with certainty. The mass of educated individuals and the mass of skilled workers remain constant over time. The income of a skilled worker remains constant at $m_{st} \equiv m_s^*$.

- (b) If m_{st} < max{a₁(δ), a₂(δ)}, at least one type of workers invest with probability less than
 1. The mass of educated individuals and that of skilled workers decrease over time. Their income increases over time and converges to some m^{*}_s ≥ max{a₁(δ), a₂(δ)}.
- (c) Steady states are ranked in inequality higher the m_s^* , higher is the inequality.
- (d) The inequality at the least unequal steady state (weakly) increases with decrease in the degree of child affinity.
- 3. When the degree of child affinity is low.
 - (a) If $m_{st} > \underline{a}_s(\delta)$, then the mass of skilled workers decreases over time and converges to zero, and correspondingly the income of a skilled worker converges to infinity.
 - (b) If $m_{st} \leq \underline{a}_s(\delta)$, then the economy immediately reaches a steady state where all workers are unskilled.
 - (c) There is no inequality at the steady state.

We prove in Appendix C.7.

The intuitions behind multiple steady states and only pure strategies being played in any such steady state are very similar to those discussed in Section 5.1.

Next, we discuss the welfare implications of the behavioral anomalies studied so far.

6 Comparison: Implications of Behavioral Trap & Behavioral Bias on Welfare

We analyze the implications of behavioral anomaly focusing on two aspects:

- (i) Distortions due to over and under investment:¹¹ We address two types of distortions:
- (a) A parent *regrets* for over or under investing *ex post*, that is if she had believed the true probability she would have not under or over invested respectively. Not Educated workers never invest, so we mainly focus on educated workers.
- (b) In the benchmark case, at any equilibrium where the unskilled workers invest, the skilled workers invest with probability 1. However, when both behavioral trap and behavioral bias are present, at an equilibrium, the Skilled workers may under invest due to the *over* investment of Educated & Unskilled workers. We explore this *crowding out* of investment due to *over* investment.

(ii) *Poverty Trap and Inequality at the steady states*: We discuss the existence of the poverty trap and compare the mass of families under poverty trap whenever possible. We compare the inequality at the 'least unequal steady states' with that at the unique steady at the

¹¹Observe, a child always prefers to be educated. We analyze from the parent's point of view and do not take her child's point of view into account.

benchmark case, whenever possible. Even when the inequalities are equal, we can rank in terms of *opportunities* per se. Though this does not arise due to any external constraint but due to internal constraint.

6.1 Implications of Behavioral Trap Only

We start with the comparison between the benchmark case and the case where only Not Educated workers are imprisoned in a behavioral trap.

(i) Distortions: We show in the next observation that no educated worker under invests.

Observation 8. 1. When the degree of child affinity is not low,

- (a) at any $m_{st} \geq 1$ the skilled workers invest with the same probability in both the cases
- (b) with behavioral trap Educated & Unskilled workers invest with strictly higher probability than that in the benchmark case when the degree of child affinity is moderate, the state variable is higher than $\underline{b}_u(\delta)$, and the mass of Not Educated workers is positive, otherwise the probabilities are equal.
- 2. When the degree of child affinity is low, all educated workers invest with the same probability in the benchmark case and in the case with behavioral trap.

We prove this in Appendix D.1.

The intuition behind the Educated & Unskilled workers investing with strictly higher probability under some parametric condition when there is a behavioral trap, goes back to the discussion we had just after Proposition 3 – why it is to possible to have $\rho_{ut} = 1$ when the degree of child affinity is moderate though $\lambda_{ut} < 1$. When the degree of child affinity is affinity, for all other cases, there are too many Skilled workers in the economy, such that no unskilled workers invest. Hence, the existence of a behavioral trap does not affect the equilibrium strategies. When the degree of child affinity is high, all educated workers invest with certainty and when that degree is low no unskilled worker invests and the Skilled workers invest with the same probability in both the cases. However, there is no regret of any educated worker – they are investing believing the true probability. The skilled workers invest with the same probability in both the cases, so there is no problem of crowding out also.

(ii) *Poverty Trap and Inequality at the steady states*: When the degree of child affinity is not low, there is no poverty trap in the benchmark case, whereas with a behavioral trap, whenever an economy starts with a positive mass of Not Educated workers, there is a poverty trap. When the degree of child affinity is low, then in both the cases, there is a poverty trap. In fact, at the steady state all adults of such an economy work as unskilled workers. We compare the steady states at various degrees of child affinity and find the following

Observation 9. When the degree of child affinity is not low, the inequality at the steady state is (weakly) higher when there is a behavioral trap.

We prove this in Appendix D.2.

Intuitively, the inequality at the steady state of the benchmark case and that at the 'least unequal steady state' are equal because only at that steady state there are enough educated workers who invest such that the income of a Skilled worker is $A\beta^{-(1-\phi)}$ or $\beta^{-(1-\phi)}\underline{b}_u(\delta)$ depending on the child affinity is high or moderate respectively. In any other steady state under behavioral trap, there are too few educated workers to invest, so the income of Skilled workers is higher, and hence, the inequality is higher.

Finally, observe, even though the inequality under the 'least unequal steady state' and that under the unique steady of the benchmark case are equal. When the degree of child affinity is moderate and the mass of Not Educated workers is positive (but not that large that 'least unequal steady state' is not achieved), we, clearly, can rank the 'least unequal steady state' and the unique steady of the benchmark case in terms of opportunity. In the benchmark case, any family can earn the higher income, earned by a Skilled worker, with a positive probability at any period, but when there is a behavioral trap, only the educated families have that opportunity. Observe, this is not due to any external constraint, but internal constraint. However, we cannot Pareto rank. As, under the behavioral trap case, here, the probability with which an Educated & Unskilled worker invests at the steady state is strictly higher than the probability with which an unskilled worker invests in the benchmark case. And, the adult of all educated family becomes an unskilled worker with a positive probability. Therefore, the probability with which a positive mass of families earn that higher income is higher than the benchmark case.

6.2 Implications of Behavioral Trap & Behavioral Bias

Now, we analyze the welfare implications when educated workers are also biased and the Not Educated workers are in a behavioral trap by comparing the findings of Section 5.2 with those of Section 4 as well as Section 5.1. Since the Not Educated workers never invest, so the under investment or over investment of the biased educated workers must be in comparison to the case where the educated workers are not biased (and the Not Educated workers do not invest). Moreover, the difference in the mass of families under the poverty trap when the educated workers are biased vis-a-vis when they are not attributes to the additional effect of their over or under investment due to their bias on non-investment of the Not Educated workers. Finally recall, the 'least unequal steady state' under behavioral trap coincide with the unique steady state of the benchmark case. So, the difference in inequality at the unique steady state of the benchmark case with that at the 'least unequal steady state' of the case where both behavioral trap and behavioral bias are present, is due to the additional effect of bias of the educated workers. Before these analyses, we cumulate the thresholds of child affinity defined in Section 4 and Section 5.2.

Observation 10. $0 < \underline{\delta} < \overline{\delta} < \delta_a$.

We prove this ranking in Appendix D.3.

(i) *Distortions*:

For huge child affinity, .e. $\delta \geq \delta_a$, all parents invest with certainty in the benchmark case, and all *educated* parents invest with certainty in the behavioral trap economy. The addition of behavioral bias, makes skilled workers over confident and lowers their incentives

to invest with certainty. Now, skilled workers invest with certainty only for skilled incomes greater than $a_2(\delta)$. Thus, behavioral bias produces only one distortion – for skilled income less that $a_2(\delta)$ skilled workers under invest. There is no over investment or crowding out – as all workers who believe the true probability of becoming a skilled worker β would invest with certainty.

When the degree of child affinity is high, i.e. $\delta \in [\bar{\delta}, \delta_a)$, investments in the benchmark and the behavioral trap are the same as the previous case with huge child affinity. With the inclusion of behavioral bias in educated workers, now all educated workers invest with certainty only for sufficiently high state variable, i.e. greater than max{ a_1, a_2 }. Now, both educated-unskilled and skilled workers can under invest at lower state variable. And, here also there is no over investment or crowding out because of the reason discussed above.

When the degree of child affinity is moderate, we have noted in Observation 8, Educated & Unskilled workers invest with (weakly) higher probability in the behavioral trap model compared to the benchmark case. When educated parents have behavioral bias, they may over or under invest. Let us consider when $m_{st} \geq \min\{a_1(\delta), a_2(\delta)\}$. The Educated & Unskilled workers underinvest or overinvest depending on whether $b_u(\delta)$ is lower or higher than $a_1(\delta)$. The Skilled workers underinvest or overinvest depending on whether $b_s(\delta)$ is lower or higher than $a_2(\delta)$. In this case, there is crowding out when $m_{st} \in [\max\{a_1(\delta), \bar{b}_s(\delta)\}, \min\{a_2(\delta), \bar{b}_u(\delta)\})$ – if the Educated & Unskilled workers had not overinvested, the Skilled workers would have invested with a higher probability. However, there is no crowding out when $m_{st} \in [\bar{b}_u(\delta), a_2(\delta))$ – the Educated & Unskilled workers do not over invest, the Skilled workers under invest because of their bias. Now, let us consider when $m_{st} < \min\{a_1(\delta), a_2(\delta)\}$. In general, Educated & Unskilled workers under invest (or over) invest at any m_{st} where γ_{ut} is respectively lower (or higher) than ρ_{ut} . Similarly, Skilled workers under (or over) invest as γ_{st} is lower (or higher) than ρ_{st} . Ex post, they regret for this under (or over) investment. There is crowding out of investment whenever $\gamma_{ut} > \rho_{ut}$ and $\gamma_{st} < \rho_{st}$.

When the degree of child affinity is low $\delta < \underline{\delta}$, unskilled workers never invest in any of the three models. The investment decision of a Skilled worker is identical under the benchmark case and the case with a behavioral trap. When both behavioral trap and behavioral trap are present, the Skilled workers over invest or under invest depending on $a_6(\delta)$ is lower or greater than $\bar{b}_s(\delta)$ respectively. Precisely, Skilled workers over invest at $m_{st} \in [a_6(\delta), \bar{b}_s(\delta))$ and under invests at $m_{st} \in [\bar{b}_s(\delta), a_6(\delta))$. There is no crowding out as no unskilled workers invest.

(ii) Poverty Trap and Inequality at the steady state: When the degree of child affinity is high and the economy has only educated workers, then there is no poverty trap in the benchmark case or the only behavioral trap case. However, when both behavioral trap and behavioral bias are present, there is a poverty trap for the parametric condition, $\max\{a_1(\delta), a_2(\delta)\} > A\beta^{-(1-\phi)}$.

We know that the existence of behavioral trap creates a poverty trap which did not exist in the benchmark case. In addition, the steady state income inequality is weakly higher in the behavioral trap. When the degree of child affinity is moderate, the behavioral bias may increase or decrease this steady state income inequality. In the presence of both behavioral trap and behavioral bias, the mass of families under poverty trap could be higher, equal or lower compared to the case when only behavioral trap exists, depending on $\beta(\underline{b}_u(\delta)/A)^{-(1-\phi)} \geq \max\{a_1(\delta), a_2(\delta)\}$. When the degree of child affinity is low, in the steady stated and across all the three scenarios, all families are in a poverty trap.

Due to behavioral bias of the educated workers, there are both over and under investments, so we find the following

- **Observation 11.** (i) When the degree of child affinity is high $\delta \geq \overline{\delta}$, the steady state inequality in the benchmark case is equal to that at the 'least unequal steady state' when both behavioral trap and behavioral bias are present only if the economy starts with all educated workers and $\max\{a_1(\delta), a_2(\delta)\} \leq A\beta^{-(1-\phi)}$. Otherwise, the inequality is lower under the benchmark case.
 - (ii) When the degree of child affinity is moderate $\delta \in (\underline{\delta}, \overline{\delta})$, the inequality at the unique steady state of the benchmark case may be higher, equal or lower than that at the 'least unequal steady state' when both behavioral trap and behavioral bias are present depending on $\beta(\underline{b}_u(\delta)/A)^{-(1-\phi)} \geq \max\{a_1(\delta), a_2(\delta)\}$ respectively.
- (iii) When the degree of child affinity is low, there is no inequality at the steady states in both the cases.

Again, observe, even when the inequalities are equal, like in the only behavioral trap case, we can rank the steady states in terms of 'opportunity'. The inequality at the 'least unequal steady state' when both behavioral trap and behavioral bias are present can be lower than that when only behavioral trap is present (which, recall, is the same as the inequality at the unique steady state at the benchmark case) only when the Educated & Unskilled workers over invest due to their bias. Though they regret *ex post*, but this provides opportunity to a higher mass of families to become rich at the steady state.

7 Appendices

A Appendix for Benchmark Case

A.1 Proof of Observation 1.

In an economy with both skilled and unskilled workers, $0 < L_{st} < 1$. Since $0 < \phi < 1$, $L_{st}^{-(1-\phi)} > 1$. Thus, $m_{st} = AL_{st}^{-(1-\phi)} \ge 1 = m_{ut}$.

A.2 Proof of Lemma 1.

(a) A skilled worker invests if and only if

$$\delta\left[\frac{[\beta m_{st+1} + (1-\beta)]^{\sigma}}{\sigma} - \frac{1}{\sigma}\right] \ge \frac{m_{st}^{\sigma}}{\sigma} - \frac{(m_{st} - \bar{s})^{\sigma}}{\sigma}$$
(A.1)

An unskilled parent invests if and only if

$$\delta\left[\frac{[\beta m_{st+1} + (1-\beta)]^{\sigma}}{\sigma} - \frac{1}{\sigma}\right] \ge \frac{1}{\sigma} - \frac{(1-\bar{s})^{\sigma}}{\sigma}$$
(A.2)

Let us define y(x) such that

$$y(x) = \frac{x^{\sigma}}{\sigma} - \frac{(x-\bar{s})^{\sigma}}{\sigma}, \quad x > 1 > \bar{s}.$$

Since $\sigma < 0$ it implies y'(x) < 0 and y''(x) > 0. Since $m_{st} > m_{ut}$, it implies $y(m_{st}) < y(m_{ut})$. Thus if

$$\delta\left[\frac{\left[\beta m_{st+1} + (1-\beta)m_{ut+1}\right]^{\sigma}}{\sigma} - \frac{m_{ut+1}^{\sigma}}{\sigma}\right] \ge \frac{m_{ut}^{\sigma}}{\sigma} - \frac{(m_{ut} - \bar{s})^{\sigma}}{\sigma} = \frac{1}{\sigma} - \frac{(1-\bar{s})^{\sigma}}{\sigma}$$

then

$$\delta\left[\frac{[\beta m_{st+1} + (1-\beta)m_{ut+1}]^{\sigma}}{\sigma} - \frac{m_{ut+1}^{\sigma}}{\sigma}\right] > \frac{m_{st}^{\sigma}}{\sigma} - \frac{(m_{st} - \bar{s})^{\sigma}}{\sigma}$$

Hence, if an unskilled worker invests in her child's education with positive probability, then a skilled worker invest in her child's education with certainty.

(b) Recall (3), a worker of type j, where $j \in \{u, s\}$, invests with probability λ_{jt}

$$\delta\left[\frac{\left[\beta^{\phi}A[\lambda_{st}L_{st}+\lambda_{ut}(1-L_{st})]^{-(1-\phi)}+1-\beta\right]^{\sigma}}{\sigma}-\frac{1}{\sigma}\right] \geq \frac{m_{jt}^{\sigma}}{\sigma}-\frac{(m_{jt}-\bar{s})^{\sigma}}{\sigma}.$$

where the inequality binds for j^{th} type when $\lambda_{jt} \in (0, 1)$. From part (a), we also know $\lambda_{ut} > 0$ only when $\lambda_{st} = 1$.

Now, as m_{st} increases, the utility cost of investment, i.e., the RHS of the above inequality decreases for skilled workers and remains the same for unskilled workers. Increase in m_{st} implies decrease in L_{st} . We show, for given $\langle \lambda_{ut}, \lambda_{st} \rangle$, the benefit, i.e. the L.H.S. of the above inequality (weakly) increases with decrease in L_{st} :

$$-\delta(1-\phi)\beta^{\phi}A\left[\beta^{\phi}A\left[\lambda_{st}L_{st}+\lambda_{ut}(1-L_{st})\right]^{-(1-\phi)}+1-\beta\right]^{\sigma-1}\cdot\left[\lambda_{st}L_{st}+\lambda_{ut}(1-L_{st})\right]^{-(2-\phi)}\cdot\left[\lambda_{st}-\lambda_{ut}\right]\leq0.$$

where the above inequality is coming from the fact that $\lambda_{st} \geq \lambda_{ut}$.

Hence, with increase in the income of the skilled worker at period t, the benefit of investment (weakly) increases and the utility cost of investment (weakly) decreases. Hence, the probability of investment must increase (weakly).

A.3 Proof of Observation 2.

 $\underline{\delta} = (1 - \bar{s})^{\sigma} - 1 \text{ and } \bar{\delta} = \frac{(1 - \bar{s})^{\sigma} - 1}{1 - [A\beta^{\phi} + 1 - \beta]^{\sigma}}.$

Clearly, $\underline{\delta} = (1 - \overline{s})^{\sigma} - 1 > 0.$

Since $A \ge 1$ and $\beta \in (0,1)$ we get that $(A\beta^{\phi} + 1 - \beta)$ is a weighted average of $A\beta^{-(1-\phi)}$ and 1. Hence, $A\beta^{\phi} + 1 - \beta \ge 1$ or $0 < (A\beta^{\phi} + 1 - \beta)^{\sigma} \le 1$. It follows,

$$(1-\bar{s})^{\sigma} - 1 < \frac{(1-\bar{s})^{\sigma} - 1}{1 - [A\beta^{\phi} + 1 - \beta]^{\sigma}} \qquad \underline{\delta} < \bar{\delta}.$$

A.4 Formal Expression for the Definition 2

(i) At $\underline{b}_s(\delta)$ a *skilled* worker is indifferent between investing and not investing, when no other worker invests. Thus, $N_{et+1} = 0$, $L_{st+1} = 0$, and $m_{st+1} \to \infty$ and it must be that

$$\delta \left[\frac{[\beta m_{st+1} + (1-\beta)]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right] = \frac{\underline{b}_s(\delta)^{\sigma}}{\sigma} - \frac{(\underline{b}_s(\delta) - \bar{s})^{\sigma}}{\sigma}$$
$$\Rightarrow \quad \underline{b}_s(\delta): \quad \underline{b}_s(\delta)^{\sigma} - (\underline{b}_s(\delta) - \bar{s})^{\sigma} + \delta = 0.$$
(A.3)

(ii) At $\bar{b}_s(\delta)$ a *skilled* worker is indifferent between investing and not investing, when all other skilled worker invest with probability 1 and no unskilled worker invests. Thus, $L_{st+1} = \beta L_{st}, m_{st+1} = A L_{st+1}^{-(1-\phi)} = \beta^{-(1-\phi)} \bar{b}_s(\delta)$ and it must be that

$$\bar{b}_s(\delta): \quad \delta\left[\frac{[\beta^{\phi}\bar{b}_s+1-\beta]^{\sigma}}{\sigma}-\frac{1}{\sigma}\right] = \frac{\bar{b}_s^{\sigma}}{\sigma}-\frac{(\bar{b}_s-\bar{s})^{\sigma}}{\sigma}.$$
 (A.4)

(iii) At $\underline{\mathbf{b}}_{\mathbf{u}}(\delta)$ a *unskilled* worker is indifferent between investing and not investing, when all skilled worker invest with probability 1 and no other unskilled worker invests. Thus,

 $L_{st+1} = \beta L_{st}, m_{st+1} = AL_{st+1}^{-(1-\phi)} = \beta^{-(1-\phi)} \underline{b}_u(\delta)$ and it must be that

$$\underline{b}_{u}(\delta): \quad \delta\left[\frac{[\beta^{\phi}\underline{b}_{u}(\delta)+1-\beta]^{\sigma}}{\sigma}-\frac{1}{\sigma}\right] = \frac{1}{\sigma}-\frac{(1-\bar{s})^{\sigma}}{\sigma}.$$
(A.5)

A.5 Proof of Lemma 2.

(i) This is immediate from the definition.

(ii) We get this by differentiating equations (A.3), (A.4) and (A.5) with respect to δ .

(iii)(a) It is sufficient to show show that $\underline{b}_u(\delta) < 1$ at $\delta = \overline{\delta}$.

$$\delta \left[\frac{[\beta^{\phi}\underline{b}_{u}(\delta) + 1 - \beta]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right] = \frac{1}{\sigma} - \frac{(1 - \bar{s})^{\sigma}}{\sigma}$$
$$\Rightarrow \quad (1 - \bar{s})^{\sigma} - 1 = \left[(1 - \bar{s})^{\sigma} - 1 \right] \left[\frac{[\beta^{\phi}\underline{b}_{u}(\delta) + 1 - \beta]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right] \quad \Rightarrow \quad \underline{b}_{u}(\delta) = \beta^{1 - \phi} < 1.$$

(iii)(b) We now show that $\bar{b}_s(\delta) > 1$ if and only if $\delta < \bar{\delta}$.

 $\bar{b}_s(\delta) > 1$ if and only if the expected net benefit (and utility cost) from the investment of a skilled parent is higher (and lower respectively) at $\bar{b}_s(\delta)$ than at 1, when all other skilled workers are investing and no unskilled worker is investing. That is, $\bar{b}_s(\delta) > 1$ if and only if

$$\delta\left[\frac{[\beta^{\phi}+1-\beta]^{\sigma}}{\sigma}-\frac{1}{\sigma}\right]<\frac{1}{\sigma}-\frac{(1-\bar{s})^{\sigma}}{\sigma}\quad\Rightarrow\quad\delta<\frac{(1-\bar{s})^{\sigma}-1}{1-[\beta^{\phi}+1-\beta]^{\sigma}}\equiv\bar{\delta}.$$

(iii)(c) We show if and only if $\delta \in [0, \underline{\delta})$, then $\underline{b}_s(\delta) > 1$. It follows directly from (A.3).

That $\underline{b}_u(\delta) = \infty$ when $\delta \in [0, \underline{\delta})$, also follows directly from (A.5). Hence, proved. \Box

A.6 Proof of Proposition 1

1. Consider any $\delta > \overline{\delta}$, from (3) it can be seen that $\gamma_{jt} = 1$ is the strictly dominating strategy for j^{th} type of worker, where $j \in \{u, s\}$. When $\delta = \overline{\delta}$, similarly, it can be seen that $\gamma_{st} = 1$ is the strictly dominating strategy for a skilled worker and $\gamma_{ut} = 1$ is weakly dominating strategy for an unskilled worker. Now, observe again from (3), if a positive mass of unskilled worker plays any strategy other than $\gamma_{ut} = 1$, then such an unskilled worker has an incentive to deviate and play $\gamma_{ut} = 1$. Therefore, $\langle 1, 1 \rangle$ is a unique equilibrium $\forall \delta \geq \overline{\delta}$.

2. Now consider the case $\delta \in [\underline{\delta}, \overline{\delta})$.

From Lemma 1, we have that in any equilibrium where $\lambda_{ut} > 0$, $\lambda_{St} = 1$. Now, from (3), it can be seen that for any $m_{st} \ge 1$, at $\langle 1, 1 \rangle$, the benefit from investment of an unskilled worker is strictly lower than her cost of investment. So, she has an incentive to deviate. Hence, $\langle 1, 1 \rangle$ cannot be an equilibrium.

2. (a), (b) and (c) Come directly from the definitions of $\underline{b}_u(\delta)$, $\overline{b}_s(\delta)$. And from the Observation 2 that $\underline{b}_s(\delta) \leq 1$. Finally observe when $\lambda_{ut} \in (0, 1)$, an unskilled worker must be indifferent between investing and not investing, that is (3) must bind, when all skilled worker invests with a positive probability and all other unskilled worker invests with that λ_{ut} , otherwise, she would have an incentive to deviate – if the benefit is strictly higher then she would invest with probability 1 and if the cost is higher then she would not invest. Similar, when $\lambda_{st} \in (0, 1)$, (3) must bind.

3. Now consider the case $\delta < \underline{\delta}$.

Again it can be seen from (3) that at $\langle \lambda_{ut}, 1 \rangle$ where $\lambda_{ut} > 0$, the benefit from investment of an unskilled worker is strictly lower than her cost of investment. So, she has an incentive to deviate. Hence, there does not exist any equilibrium where $\lambda_{ut} > 0$.

3. (a), (b) and (c) Now directly follows from the definitions of $\bar{b}_s(\delta)$ and $\underline{b}_s(\delta)$. Following the argument above, it can be shown that when $\lambda_{st} \in (0, 1)$ $\lambda_{st} \in (0, 1)$, (3) must bind.

A.7 Proof of Proposition 2

1. (a) From Proposition 1 1., we have that all parents invest with certainty. So, the economy immediately enters into a steady state, the mass of skilled worker is $L_s^* = \beta \cdot 1 = \beta$ and the income of a skilled worker is $A(L_s^*)^{-(1-\phi)} = A\beta^{-(1-\phi)}$.

Now we argue at the steady state, the probability with which an adult works as a skilled worker is β :

$$\begin{split} \beta \cdot [\lambda_s^* \cdot \mbox{ the probability that her parent was a skilled worker} \\ &+ \lambda_u^* \cdot \mbox{ the probability that her parent was an unskilled worker }] \\ = &\beta \cdot 1 \\ = &\beta \end{split}$$

where the second equality comes from $\lambda_s^* = \lambda_u^* = 1$.

1. (b) Observe $\langle 0,1 \rangle$ or $\langle (0,1),1 \rangle$ cannot be the equilibrium strategy at any steady state because in those cases, the mass of skilled workers decreases over time. Also observe from Proposition 1, those strategies are equilibria only when m_{st} is lower than $\underline{b}_u(\delta)$ or the mass of skilled workers is higher than $\beta (\underline{b}_u(\delta)/A)^{-\frac{1}{1-\phi}}$.

Hence, if an economy starts with a mass of skilled workers higher than $\beta \left(\underline{b}_u(\delta)/A\right)^{-\frac{1}{1-\phi}}$, then only skilled workers invest with a positive probability and hence, the mass of skilled workers decreases over time and their income increases over time and reaches $\underline{b}_u(\delta)$.

Now, we show that λ_{ut} is such that the economy enters into a steady state at t + 1. For that consider, again, the incentive constraint of an unskilled worker when when all other unskilled workers are investing with probability λ_{ut} and all skilled workers are investing with

certainty

$$\frac{(m_{ut}-\bar{s})^{\sigma}}{\sigma} + \delta \frac{[\beta m_{st+1} + (1-\beta)m_{ut+1}]^{\sigma}}{\sigma} = \frac{m_{ut}^{\sigma}}{\sigma} + \delta \frac{m_{ut+1}^{\sigma}}{\sigma}$$

At the steady state, $L_{st+1} = L_{st}$ which implies

$$\beta(\lambda_{ut} + (1 - \lambda_{ut})L_{st}) = L_{st+1} = \left[\frac{1}{\beta A} \left[\left[\frac{1 + \delta - (1 - \bar{s})^{\sigma}}{\delta}\right]^{\frac{1}{\sigma}} - (1 - \beta) \right] \right]^{-\frac{1}{1 - \phi}} \equiv \beta \left(\frac{\underline{b}_u(\delta)}{A}\right)^{-\frac{1}{1 - \phi}}$$
$$\Rightarrow \quad \lambda_{ut} = \frac{\left(\underline{b}_u(\delta)/A\right)^{-\frac{1}{1 - \phi}} - L_{st}}{1 - L_{st}}$$

Observe, $\underline{b}_u(\delta)$ is time independent, hence L_{st+1} is time independent. So, if an economy is such that $\lambda_{ut} = \frac{(\underline{b}_u(\delta)/A)^{-\frac{1}{1-\phi}} - L_{st}}{1 - L_{st}}$ and $\lambda_{st} = 1$, then the economy is at a steady state at t + 1. At the steady state, mass of skilled worker $L_s^* \equiv \beta (\underline{b}_u(\delta)/A)^{-\frac{1}{1-\phi}}$, wage of a skilled worker $m_s^* \equiv \beta^{-(1-\phi)}\underline{b}_u(\delta)$ and $\lambda_u^* \equiv \frac{(\underline{b}_u(\delta)/A)^{-\frac{1}{1-\phi}} - L_s^*}{1 - L_s^*}$.

To check that indeed this is a steady state, we need $\lambda_u^* \in (0, 1)$. Now, $\lambda_u^* < 1$ because the degree of child affinity is not high, i.e. $\delta < \overline{\delta}$. And, $\lambda_u^* > 0$ because $m_s^* \equiv \beta^{-(1-\phi)} \underline{b}_u(\delta) > \underline{b}_u(\delta)$. Hence, at the steady state, the mass of skilled worker is $L_s^* \equiv \beta (\underline{b}_u(\delta)/A)^{-\frac{1}{1-\phi}}$, the wage of a skilled worker $m_s^* \equiv \beta^{-(1-\phi)} \underline{b}_u(\delta)$ and an unskilled worker invests with probability $\lambda_u^* \equiv \frac{(\underline{b}_u(\delta)/A)^{-\frac{1}{1-\phi}} - L_s^*}{1 - L_s^*}$ whereas a skilled worker invests with probability 1.

At the steady state, the probability with which an adult works as a skilled worker is

 $\beta \cdot [\lambda_s^* \cdot \text{ the probability that her parent was a skilled worker} + \lambda_u^* \cdot \text{ the probability that her parent was an unskilled worker]} <1.$

where the inequality comes from $\lambda_u^* < 1$. Also, observe differentiating λ_u^* with respect to $\underline{b}_u(\delta)$, we get that λ_u^* strictly increases with decrease in $\underline{b}_u(\delta)$ and $\underline{b}_u(\delta)$ strictly decreases with increase in δ , i.e., as δ decreases λ_u^* strictly decreases and $\lambda_s^* = 1$. Hence the result.

1. (c) When the degree of child affinity is high, the income of a skilled worker is $A\beta^{-(1-\phi)}$ and that of an unskilled worker is 1. So, the inequality is the same $\forall \delta \geq \bar{\delta}$. When the degree of child affinity is moderate, the income of a skilled worker is $\beta^{-(1-\phi)}\underline{b}_u(\delta)$ and that of an unskilled worker is 1. Now, $\underline{b}_u(\delta)$ strictly decreases with increase in δ . So, the difference between the income of a skilled worker and that of an unskilled worker decreases

2. (a) From Part 3. of Proposition 1., we have that when $\delta < \underline{\delta}$, then no unskilled workers

with increase in δ . Hence, the inequality increases with decrease in δ .

invest at any m_{st} . Moreover, when $m_{st} > \underline{b}_s(\delta)$, then skilled workers invest with a positive probability. So, the mass of educated workers and hence, the mass of skilled workers decrease over time and converge to zero, whereas the income of a skilled worker increases over time and converges to infinity.

2. (b) From Part 3. of Proposition 1., we have $m_{st} \leq \underline{b}_s(\delta)$, no parents invest. So, the economy is is in a steady state where no parent invests and all workers are unskilled.

3. (c) All adults work as unskilled workers and earn 1. Hence, the result.

B Appendix for Bias via Education: Behavioral Trap

B.1 Formal Statement of Definition 3

Recall the definition of $\bar{\mathbf{b}}_{\mathbf{u}}(\delta)$. The minimum value of the state variable (m_{st}) at period t, at which Educated & Unskilled workers invest with certainty, i.e. $\rho_{ut} = 1$ if and only if $m_{st} \leq \bar{b}_u(\delta)$. From Lemma 1 (a), we know at $\bar{b}_u(\delta)$, all skilled workers invest with certainty.

So, at $b_u(\delta)$ an Educated & Unskilled worker is just indifferent between investing and not investing, when all unskilled workers invest with certainty. Observe, at any $m_{st} \geq \bar{b}_u(\delta)$, the mass of educated worker at t + 1 would be N_{et} . So, $m_{st+1} = m_{st}$.

$$\bar{b}_u(\delta): \quad \delta\left[\frac{[\beta\bar{b}_u(\delta)+1-\beta]^{\sigma}}{\sigma}-\frac{1}{\sigma}\right] = \frac{1}{\sigma}-\frac{(1-\bar{s})^{\sigma}}{\sigma}.$$
(A.6)

B.2 Proof of Observation 5

(i) We get this by differentiating equation (A.6).

(ii) Recall $\bar{\delta} \equiv \frac{(1-\bar{s})^{\sigma}-1}{1-(A\beta^{\phi}+1-\beta)^{\sigma}}$ and from the definition of $\bar{b}_u(\delta)$ we have $\bar{b}_u(\delta) = A\beta^{-(1-\phi)}$.

Suppose $\delta \in (\underline{\delta}, \overline{\delta})$. From the definition of $\underline{b}_u(\delta)$ and $\overline{b}_u(\delta)$ we have $\overline{b}_u(\delta) = \beta^{-(1-\phi)}\underline{b}_u(\delta)$. That $\overline{b}_u(\delta) = \infty$ when the degree of child affinity is low follows directly from (A.6).

(iii) Now, it is clear that $\underline{b}_u(\delta) = \beta^{1-\phi} \overline{b}_u(\delta) < \overline{b}_u(\delta)$.

B.3 Proof of Proposition **3**

- 1. See Observation 3.
- 2. Consider (4), $\rho_{ut} = 1$ if

$$\delta\left[\frac{\left[\beta^{\phi}A[(1-\beta)N_{et}+\beta N_{et}]^{-(1-\phi)}+1-\beta\right]^{\sigma}}{\sigma}-\frac{1}{\sigma}\right] \geq \frac{1}{\sigma}-\frac{(1-\bar{s})^{\sigma}}{\sigma}.$$

Now, the benefit, i.e. the L.H.S. increases with decrease in N_{et} and the maximum value N_{et} can take is 1. Similarly, the L.H.S. increases with increase in δ . So, to prove this, it sufficient to show that L.H.S. is no less than R.H.S. at $N_{et} = 1$ and $\delta = \overline{\delta}$:

$$\bar{\delta}\left[\frac{\left[\beta^{\phi}A+1-\beta\right]^{\sigma}}{\sigma}-\frac{1}{\sigma}\right] \geq \frac{1}{\sigma}-\frac{(1-\bar{s})^{\sigma}}{\sigma}.$$

Now, from the definition of $\bar{\delta}$, it can be seen that L.H.S. is equal to R.H.S. Hence, $\rho_{ut} = 1$ $\forall N_{et} \in [0, 1]$ and $\delta \geq \bar{\delta}$.

3. (a),(b),(c),(d) Coming from the definitions of $\bar{b}_u(\delta)$, $\underline{b}_u(\delta)$, $\underline{b}_s(\delta)$, $\underline{b}_s(\delta)$, and Lemma 2 (iii).

4. (a),(b),(c) Coming from the definitions of $\bar{b}_u(\delta)$, $\bar{b}_s(\delta)$, $\underline{b}_s(\delta)$, and Lemma 2 Part (iii).

B.4 Proof of Proposition 4

1. Not Educated workers never invest, so if an economy starts with a positive mass of Not Educated workers then there will be a poverty trap.

When $\delta < \overline{\delta}$, even if the economy starts with all educated workers, Educated & Unskilled workers invest with probability less than 1. Hence, there will be a positive mass of Not Educated workers in the next period. Following the argument above, we can see that there will be poverty trap in the economy.

Thus, in any economy, there exists a poverty trap almost always.

2. (a) From Proposition 3., we have that when $\delta \geq \overline{\delta}$, there is a unique steady state where all educated workers invest. If the mass of Not Educated workers is zero then the steady state income of a skilled worker would be $\overline{b}_u(\overline{\delta})$. If the mass of Not Educated workers is positive, then that would be strictly higher than $\overline{b}_u(\overline{\delta})$. All the educated workers invest forever from the beginning. Hence, the result.

2. (b) From Proposition 3., we have that when the degree of child affinity is moderate and $m_{st} \geq \bar{b}_u(\delta)$, all educated workers invest. So, the mass of educated workers, and hence the mass of skilled workers and their income remain constant over time. Therefore, any $m_{st} \geq \bar{b}_u(\delta)$ is a steady state.

2. (c) From Proposition 3., we have that when $m_{st} < \bar{b}_u(\delta)$, Educated & Unskilled workers invest with probability less than 1. Hence, the mass of educated workers, and hence the mass of skilled workers decrease over time. Since, the income of a skilled worker is inversely related to the mass of skilled workers, this implies the income of a skilled worker increases over time. This happens till at some $t m_{st} \ge \bar{b}_l \delta$.

Then, the parametric condition satisfies that described in Part 2. (b) of this proposition. Hence, the steady state. 2. (d) This is obvious – the inequality at the steady state – the difference between the income of a skilled worker and that of an unskilled worker increases with increase in the income of a skilled worker, as that of an unskilled worker is a constant.

2. (e) The difference between $\bar{b}_u(\bar{\delta})$ and income of an unskilled worker is a constant. So, when the degree of child affinity is high the inequality at the least unequal steady state is a constant. Now, $\bar{b}_u(\delta)$ increases with decrease in δ . Hence, the result.

3. This is very similar to the proof of Proposition 2 Part 2., so we skip. \Box

C Appendix for Bias via Education and Job Network

C.1 Proof of Lemma 3

Suppose not, there exists an equilibrium $\langle \gamma_{ut}, \gamma_{st} \rangle$ such that $\gamma_{st} = 0$ and $\gamma_{ut} > 0$.

We show this is not possible. The argument is as follows. $\langle \gamma_{ut}, \gamma_{st} \rangle$ is an equilibrium at any state variable m_{st} imply that (i) an Educated & Unskilled worker is indifferent between investing and not investing when all other Educated & Unskilled workers invest with probability γ_{ut} , and no Educated & Skilled worker invests, and (ii) the benefit from investment of an Educated & Skilled worker is no higher than her utility cost, when all Educated & Unskilled workers invest with probability γ_{ut} and no other Educated & Skilled worker invests. But we show that when (i) is true, (ii) cannot hold. So, an Educated & Skilled worker would have an incentive to deviate unilaterally implying $\langle \gamma_{ut}, \gamma_{st} \rangle$ cannot be an equilibrium. Formally, at equilibrium $\gamma_{ut} > 0$ and $\gamma_{st} = 0$ imply

$$\delta \left[1 - \left[\theta \beta [\theta \gamma_{ut} (1-\beta)]^{-(1-\phi)} m_{st} + 1 - \theta \beta \right]^{\sigma} \right]$$

$$\geq (1-\bar{s})^{\sigma} - 1$$

$$> (m_{st} - \bar{s})^{\sigma} - m_{st}^{\sigma}$$

$$> \delta \left[1 - \left[[1 - \theta (1-\beta)] [\gamma_{ut} (1-\beta)]^{-(1-\phi)} m_{st} + \theta (1-\beta) \right]^{\sigma} \right]$$

$$\Rightarrow (1-\theta) \geq [\gamma_{ut} (1-\beta)]^{-(1-\phi)} \left[[1 - \theta (1-\beta)] - \theta^{\phi} \beta \right] m_{st}.$$
(A.7)

Now, define a function $L(\theta) = (1-\beta)^{-(1-\phi)}(1-\theta(1-\beta)-\theta^{\phi}\beta) - 1 + \theta$.

Observe, $L(0) = (1 - \beta)^{-(1-\phi)} - 1$ and L(1) = 0. Further,

$$L'(\theta) = -(1-\beta)^{-(1-\phi)}(1-\beta+\phi\theta^{-(1-\phi)}\beta) + 1$$

$$L'(\theta) = 0 \text{ at } \theta = \left[\frac{(1-\beta)^{(1-\phi)}-(1-\beta)}{\phi\beta}\right]^{-\frac{1}{1-\phi}} > \left[\frac{1}{\phi}\right]^{-\frac{1}{1-\phi}} > 1$$

$$L''(\theta) = (1-\beta)^{-(1-\phi)}\phi(1-\phi)\beta\theta^{-(2-\phi)} > 0$$

Since $L'(\theta) < 0$ for all $\theta \in [0, 1]$ and the boundary values of $L(\theta)$ at 0 and 1 are non-negative,

 $L(\theta) > 0$ for all $\theta \in [0, 1]$. Thus,

$$(1-\beta)^{-(1-\phi)} \left[\left[1 - \theta(1-\beta) - \theta^{\phi} \beta \right] \ge 1 - \theta$$

Hence, $[\gamma_{ut}(1-\beta)]^{-(1-\phi)}[[1-\theta(1-\beta)-\theta^{\phi}\beta]m_{st} > (1-\beta)^{-(1-\phi)}[[1-\theta(1-\beta)-\theta^{\phi}\beta] \ge 1-\theta$ which contradicts (A.7).

C.2 Proof of Observation 6

We have already shown in Observation 2 that $0 < \underline{\delta}$. Here we show, $\underline{\delta} < \delta_a$. The weighted average of $(\theta(1-\beta)+\beta)^{-(1-\phi)}$ and one will be greater than one. It follows,

$$\underline{\delta} = (1-\bar{s})^{\sigma} - 1 < \frac{(1-\bar{s})^{\sigma} - 1}{1 - [\theta\beta(\theta(1-\beta) + \beta)^{-(1-\phi)} + 1 - \theta\beta]^{\sigma}} = \delta_a.$$

C.3 Formal Expressions for Definition 5

(i) Given Lemma 3, when $\gamma_{st} = 0$, γ_{ut} is also zero. Hence, for a given degree of child affinity δ , at $\underline{a}_s(\delta)$ a Skilled worker is indifferent between investing and not investing, when no other worker invests. So, from (6) we have

$$\underline{a}_s(\delta): \quad \underline{a}_s(\delta)^{\sigma} - (\underline{a}_s(\delta) - \bar{s})^{\sigma} + \delta = 0.$$
(A.8)

(ii) For a given degree of child affinity δ , at $a_6(\delta)$ a Skilled worker is just indifferent between investing and not investing, when all other Skilled workers are investing with certainty and no Educated & Unskilled worker is investing. So, from (6)

$$a_{6}(\delta): \quad \frac{\delta}{\sigma} \left[\left[\left[1 - \theta(1-\beta) \right]^{\phi} a_{6}(\delta) + \theta(1-\beta) \right]^{\sigma} - 1 \right] = \frac{a_{6}(\delta)^{\sigma} - (a_{6}(\delta) - \bar{s})^{\sigma}}{\sigma}. \quad (A.9)$$

(iii) For a given degree of child affinity δ , at $a_4(\delta)$ a Skilled worker is just indifferent between investing and not investing, when no other Skilled worker is investing and all Educated & Unskilled workers are investing with certainty. So, from (6)

$$a_4(\delta): \quad \frac{\delta}{\sigma} \left[\left[[1 - \theta(1 - \beta)](1 - \beta)^{-(1 - \phi)} a_4(\delta) + \theta(1 - \beta) \right]^{\sigma} - 1 \right] = \frac{a_4(\delta)^{\sigma} - (a_4(\delta) - \bar{s})^{\sigma}}{\sigma}.$$
(A.10)

(iv) For a given degree of child affinity δ , at $a_2(\delta)$ a Skilled worker is just indifferent between investing and not investing, when all other educated workers are investing with certainty. So, from (6)

$$a_{2}(\delta): \qquad \frac{\delta}{\sigma} \Big[\big[[1 - \theta(1 - \beta)] [1 + (1 - \theta)(1 - \beta)]^{-(1 - \phi)} a_{2}(\delta) + \theta(1 - \beta) \big]^{\sigma} - 1 \Big] \\ = \frac{a_{2}(\delta)^{\sigma} - (a_{2}(\delta) - \bar{s})^{\sigma}}{\sigma}.$$
(A.11)

(v) For a given degree of child affinity δ , at $a_5(\delta)$ an Educated & Unskilled worker is just indifferent between investing and not investing, when all Skilled workers are investing and no other Educated & Unskilled worker is investing. So, from (5)

$$a_5(\delta): \frac{\delta}{\sigma} \left[\left[\theta \beta^{\phi} a_5(\delta) + 1 - \theta \beta \right]^{\sigma} - 1 \right] = \frac{1 - (1 - \bar{s})^{\sigma}}{\sigma}.$$
 (A.12)

(vi) For a given degree of child affinity δ , at $a_3(\delta)$ an Educated & Unskilled worker is just indifferent between investing and not investing, when all other Educated & Unskilled workers are investing with certainty and no Skilled worker is investing. So, from (5)

$$a_{3}(\delta): \quad \frac{\delta}{\sigma} \left[\left[\theta \beta [\theta(1-\beta)]^{-(1-\phi)} a_{3}(\delta) + 1 - \theta \beta \right]^{\sigma} - 1 \right] = \frac{1 - (1-\bar{s})^{\sigma}}{\sigma} = 0.$$
 (A.13)

(vii) For a given degree of child affinity δ , at $a_1(\delta)$ an Educated & Unskilled worker is just indifferent between investing and not investing, when all other educated workers are investing with certainty. So, from (6)

$$a_{1}(\delta): \quad \frac{\delta}{\sigma} \Big[\big[\theta \beta [\theta(1-\beta) + \beta]^{-(1-\phi)} a_{1}(\delta) + 1 - \theta \beta \big]^{\sigma} - 1 \Big] = \frac{1 - (1-\bar{s})^{\sigma}}{\sigma}. \quad (A.14)$$

C.4 Proof of Lemma 4

i. We want to show that all income cut-offs are decreasing in δ . Consider the income-cutoff $a_6(\delta)$, which is determined by (A.9). The L.H.S. of (A.9) is increasing in a_6 and δ and the R.H.S is decreasing in a_6 . Differentiating the equation gives $da_6/d\delta < 0$. We get the same result for the other income cut-offs \underline{a}_s , a_1 , a_2 , a_3 , a_4 and a_5 by differentiating their respective equations (A.8), (A.10)–(A.14).

ii. (a) We show $a_2(\delta) > 1$. Suppose not. From the argument used in the proof of Lemma 1

$$(a_2(\delta) - \bar{s})^{\sigma} - a_2(\delta)^{\sigma} \ge (1 - \bar{s})^{\sigma} - 1 > 0$$

that is, the utility cost of investment is positive. So, it is enough to show that the benefit from investment (which satisfies the primitives of the definition) is negative:

$$\frac{\delta}{\sigma} \Big[\big[[1 - \theta(1 - \beta)] [1 + (1 - \theta)(1 - \beta)]^{-(1 - \phi)} a_2(\delta) + \theta(1 - \beta) \big]^{\sigma} - 1 \Big] < 0.$$

The following steps give us that

$$[1 + (1 - \theta)(1 - \beta)]^{-(1 - \phi)} < 1$$

$$\Rightarrow [1 - \theta(1 - \beta)][1 + (1 - \theta)(1 - \beta)]^{-(1 - \phi)}a_2(\delta) + \theta(1 - \beta) < [1 - \theta(1 - \beta)]a_2(\delta) + \theta(1 - \beta)$$

$$\leq 1 - \theta(1 - \beta) + \theta(1 - \beta)$$

$$= 1$$

ii. (b) We first show $a_2(\delta) > a_4(\delta)$. Suppose not and $a_2(\delta) \le a_4(\delta)$. We obtain the contra-

diction in two steps – the utility cost of investment at $a_2(\delta)$ is no less than that at $a_4(\delta)$ and the benefit from investment (satisfying the respective primitives of the definitions) at $a_2(\delta)$ is strictly higher than that at $a_4(\delta)$.

That the utility cost of investment at $a_2(\delta)$ is no less than that at $a_4(\delta)$

$$(a_4(\delta) - \bar{s})^{\sigma} - a_4(\delta)^{\sigma} \le (a_2(\delta) - \bar{s})^{\sigma} - a_2(\delta)^{\sigma}$$

is immediate from the argument used in Lemma 1.

Now, compare the benefits, recall, the primitives of definition of $a_4(\delta)$ and $a_2(\delta)$. For $a_4(\delta)$: all Educated & Unskilled workers invest. For $a_2(\delta)$: all educated workers invest.

$$1 + (1 - \theta)(1 - \beta) > 1 - \beta$$

$$\Rightarrow \quad \delta \Big[1 - \big[[1 - \theta(1 - \beta)] [1 + (1 - \theta)(1 - \beta)]^{-(1 - \phi)} a_2(\delta) + \theta(1 - \beta) \big]^{\sigma} \Big]$$

$$< \quad \delta \Big[1 - \big[[1 - \theta(1 - \beta)] (1 - \beta)^{-(1 - \phi)} a_4(\delta) + \theta(1 - \beta) \big]^{\sigma} \Big]$$

This gives us the desired contradiction.

Now, we show $a_2(\delta) > a_6(\delta)$. We follow similar steps and prove this by contradiction.

Suppose not and $a_2 \leq a_6$ which implies $(a_6 - \bar{s})^{\sigma} - a_6^{\sigma} \leq (a_2 - \bar{s})^{\sigma} - a_2^{\sigma}$. At the same time,

$$1 + (1 - \theta)(1 - \beta) > 1 - \theta(1 - \beta)$$

$$\Rightarrow \quad \delta \Big[1 - \big[[1 - \theta(1 - \beta)] [1 + (1 - \theta)(1 - \beta)]^{-(1 - \phi)} a_2(\delta) + \theta(1 - \beta) \big]^{\sigma} \Big]$$

$$< \quad \delta \Big[1 - \big[[1 - \theta(1 - \beta)]^{\phi} a_6(\delta) + \theta(1 - \beta) \big]^{\sigma} \Big]$$

This is a contradiction.

Finally, we show $\underline{a}_s(\delta) < a_4(\delta)$ and $\underline{a}_s(\delta) < a_6(\delta)$.

Here, we simply note that the benefits at the primitive of $\underline{a}_s(\delta)$ is higher than that at the primitive of $a_4(\delta)$, as well as that of $_6(\delta)$. Hence, to make a skilled worker indifferent, we must have $\underline{a}_s(\delta)$ lower than $a_4(\delta)$ as well as $a_6(\delta)$.

For this, observe that the benefit at the primitive of $\underline{a}_{s}(\delta)$ is infinity whereas that at the primitive of $a_{4}(\delta)$ or of $a_{6}(\delta)$ is finite. Hence, the result.

ii. (c) $a_4(\delta) > a_6(\delta)$ if and only if $\theta(1-\beta) > \beta$.

First, we show by contradiction that when $\theta(1-\beta) > \beta$ then $a_4(\delta) > a_6(\delta)$. The converse can be shown similarly which we skip.

Suppose not $\theta(1-\beta) > \beta$ and $a_4(\delta) \le a_6(\delta)$ which implies utility cost of investment at $a_4(\delta)$ is no less than that at $a_6(\delta)$:

$$(a_6(\delta) - \bar{s})^{\sigma} - a_6(\delta)^{\sigma} \le (a_4(\delta) - \bar{s})^{\sigma} - a_4(\delta)^{\sigma}$$

We show the benefit at the primitive of $a_4(\delta)$ is strictly lower than that of $a_6(\delta)$:

$$\theta(1-\beta) > \beta \implies 1-\theta(1-\beta) < 1-\beta$$

$$\Rightarrow \quad \delta \Big[1 - \big[[1-\theta(1-\beta)](1-\beta)^{-(1-\phi)} a_4(\delta) + \theta(1-\beta) \big]^{\sigma} \Big]$$

$$< \quad \delta \Big[1 - \big[[1-\theta(1-\beta)]^{\phi} a_6(\delta) + \theta(1-\beta) \big]^{\sigma} \Big]$$

Hence, the statements of both the definitions of $a_4(\delta)$ and $a_6(\delta)$ cannot simultaneously be true.

iii. (a) We show $a_1(\delta)$, $a_3(\delta)$ and $a_5(\delta)$ are finite if and only if $\delta > \underline{\delta} \equiv (1 - \overline{s})^{\sigma} - 1$. Let us first consider the income cut-off $a_1(\delta)$. We rewrite (A.14) as

$$\left[\theta\beta[\theta(1-\beta)+\beta]^{-(1-\phi)}a_1(\delta)+1-\theta\beta\right]^{\sigma}=\frac{1-(1-\bar{s})^{\sigma}+\delta}{\delta}.$$

If $\delta \leq (1-\bar{s})^{\sigma} - 1$, then R.H.S is negative and hence there does not exist any finite $a_1(\delta)$ which would satisfy the above equation. If $\delta > (1-\bar{s})^{\sigma} - 1$, then R.H.S is a positive fraction. Since L.H.S. is decreasing and convex in $a_1(\delta)$ and is bounded between $[0, (1-\theta\beta)^{\sigma}]$ for all $a_1(\delta) > 0$. Thus, there exists a finite $a_1(\delta)$ at which L.H.S. equals R.H.S.

We follow analogous reasoning and use equations (A.12) and (A.13) to show the same for the income-cutoffs $a_5(\delta)$ and $a_3(\delta)$.

iii. (b) $1 < a_1(\delta)$ if and only if $\delta < \delta_a$.

Suppose $a_1(\delta) \gtrsim 1$. Using this in (A.14) we get

$$\delta \gtrless \frac{(1-\bar{s})^{\sigma} - 1}{1 - [\theta\beta(\theta(1-\beta) + \beta)^{-(1-\phi)} + 1 - \theta\beta]^{\sigma}} \equiv \delta_a.$$

This proves the claim.

We now determine the rankings among the thresholds of the state variable which are relevant for Educated & Unskilled workers. Now, their cost of investment is independent of the state variable, so we need to compare only the benefits while ranking those thresholds. Also, we know $a_1(\delta)$, $a_3(\delta)$ and $a_5(\delta)$ are finite if and only if $\delta > \underline{\delta}$, so the following rankings are only when $\delta > \underline{\delta}$. iii. (c) First we show $a_5(\delta) < a_1(\delta)$ Comparing (A.14) and (A.12) we get,

$$\begin{split} \delta \Big[1 - \big[\theta \beta [\theta(1-\beta) + \beta]^{-(1-\phi)} a_1(\delta) + 1 - \theta \beta \big]^{\sigma} \Big] &= \delta \Big[1 - \big[\theta \beta^{\phi} a_5(\delta) + 1 - \theta \beta \big]^{\sigma} \Big] \\ \Rightarrow \quad \theta \beta [\theta(1-\beta) + \beta]^{-(1-\phi)} a_1(\delta) &= \theta \beta^{\phi} a_5(\delta) \\ \Rightarrow \quad [\theta(1-\beta) + \beta]^{-(1-\phi)} a_1(\delta) &= \beta^{-(1-\phi)} a_5(\delta) \\ \Rightarrow \quad a_1(\delta) > a_5(\delta) \quad \text{as } \theta(1-\beta) + \beta > \beta \end{split}$$

Now, we show $a_3(\delta) < a_1(\delta)$ Comparing (A.14) and (A.13), we get

$$\delta \left[1 - \left[\theta \beta [\theta(1-\beta) + \beta]^{-(1-\phi)} a_1(\delta) + 1 - \theta \beta \right]^{\sigma} \right] = \delta \left[1 - \left[\theta \beta [\theta(1-\beta)]^{-(1-\phi)} a_3(\delta) + 1 - \theta \beta \right]^{\sigma} \right]$$

$$\Rightarrow \quad \theta \beta [\theta(1-\beta) + \beta]^{-(1-\phi)} a_1(\delta) = \theta \beta [\theta(1-\beta)]^{-(1-\phi)} a_3(\delta)$$

$$\Rightarrow \quad a_1(\delta) > a_3(\delta) \quad \text{as } \theta(1-\beta) + \beta > \theta(1-\beta)$$

iii. (d) $\underline{a_3(\delta) > a_5(\delta)}$ if and only if $\theta(1-\beta) > \beta$ Comparing (A.13) and (A.12) we get,

$$\delta \Big[1 - \big[\theta \beta [\theta(1-\beta)]^{-(1-\phi)} a_3(\delta) + 1 - \theta \beta \big]^{\sigma} \Big] = \delta \Big[1 - \big[\theta \beta^{\phi} a_5(\delta) + 1 - \theta \beta \big]^{\sigma} \Big]$$

$$\Rightarrow \quad \theta \beta [\theta(1-\beta)]^{-(1-\phi)} a_3(\delta) = \theta \beta^{\phi} a_5(\delta)$$

Hence, $a_3(\delta) > a_5(\delta)$ if and only if $[\theta(1-\beta)]^{-(1-\phi)} < \beta^{-(1-\phi)} \Rightarrow \theta(1-\beta) > \beta$.

iv. First, we show $\forall \delta \leq \delta_a$, $a_4(\delta) \leq a_1(\delta)$.

Suppose not. $\exists \delta \leq \delta_a$ such that $a_1(\delta) < a_4(\delta)$. Then consider any $m_{st} \in [a_1(\delta), a_4(\delta)]$. Since, $m_{st} \geq a_1(\delta), \gamma_{ut} = 1 \ \forall \gamma_{st} = [0, 1]$. And, by the definition of $a_4(\delta), \gamma_{st}$ must be equal to zero which contradicts Lemma 3.

Now, we show
$$\forall \delta > \delta_a$$
, $a_4(\delta) < 1$.

It can again be proved by contradiction following the above argument and considering the range $m_{st} \in [1, a_4(\delta)]$.

v. (a) Comparing definitions (A.3) and (A.8) we get $\underline{b}_s(\delta) = \underline{a}_s(\delta)$.

v. (b)
$$\underline{b}_{u}(\delta) < a_{5}(\delta)$$
.

We show that the benefit from investment with behavioral anomaly at $\underline{b}_u(\delta)$ when $\theta < 1$ is strictly lower than that when $\theta = 1$, i.e.

$$\delta\left[\frac{[\theta\beta^{\phi}\underline{b}_{u}(\delta)+1-\theta\beta]^{\sigma}}{\sigma}-\frac{1}{\sigma}\right] < \delta\left[\frac{[\beta^{\phi}\underline{b}_{u}(\delta)+1-\beta]^{\sigma}}{\sigma}-\frac{1}{\sigma}\right]$$

so, $a_5(\delta) \geq \underline{b}_u(\delta) \ \forall \ \theta \in [0, 1]$, and strictly higher $\forall \ \theta < 1$.

Let $x = \theta \beta^{\phi} \underline{b}_u(\delta) + 1 - \theta \beta$. Then,

$$\frac{\partial x}{\partial \theta} = \beta^{\phi} \underline{b}_u(\delta) - \beta > 0 \qquad \text{if } \underline{b}_u > \beta^{1-\phi}.$$

Observe at $\underline{b}_u = \beta^{1-\phi}$, the benefit from investment when $\theta = 1$ is zero and cost is positive, but $\underline{b}_u(\delta)$ should be such that the benefit is equal to cost. The benefit is increasing in m_{st} , hence, $\underline{b}_u(\delta)$ must be greater than $\beta^{1-\phi}$. Hence, we retrace the steps to find $\underline{b}_u < a_5$.

C.5 Derivation of Conditions Depicted in Lemma 5

Condition $\underline{\Gamma}_s$: First, we define $\underline{\gamma}_s(m_{st})$. Then, we prove its properties

$$\underline{\gamma}_{s}(m_{st}) \begin{cases} = 0 & \text{at } m_{st} = a_{4}(\delta) \\ \in (0,1) & \forall m_{st} \in (a_{4}(\delta), a_{2}(\delta)) \\ = 1 & \text{at } m_{st} \ge a_{2}(\delta). \end{cases}$$

and $\underline{\gamma}_s(m_{st})$ is strictly increasing $\forall m_{st} \in [a_4(\delta), a_2(\delta))$. We, also, show that when $m_{st} \geq a_4(\delta)$, at any equilibrium γ_{st} is bounded below by $\underline{\gamma}_s(m_{st})$. We consider three cases – Case 1. $m_{st} \in (a_4(\delta), a_2(\delta))$, Case 2. $m_{st} \geq a_2(\delta)$ and Case 3. $m_{st} = a_4(\delta)$ one by one, and prove the relevant properties of that range.

Suppose, $m_{st} \ge a_4(\delta)$. $\underline{\gamma}_s(m_{st})$ is defined as follows. A skilled worker is indifferent between investing and not investing when all Educated & Unskilled workers invest with certainty and all other Skilled workers invest with $\gamma_s(m_{st})$. That is,

$$\delta \left[\frac{\left[\left[1 - \theta(1 - \beta) \right] \cdot \left[(1 - \beta) + \left[1 - \theta(1 - \beta) \right] \underline{\gamma}_s(m_{st}) \right]^{-(1 - \phi)} m_{st} + \theta(1 - \beta) \right]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right]$$
$$\geq \frac{m_{st}^{\sigma}}{\sigma} - \frac{(m_{st} - \bar{s})^{\sigma}}{\sigma}$$

From the definitions of $a_4(\delta)$ and $a_6(\delta)$, it is clear that $m_{st} = a_4(\delta)$, $\underline{\gamma}_s(m_{st}) = 0$, and at $m_{st} = a_2(\delta)$, $\gamma_s(m_{st}) = 1$.

Case 1. Consider any $m_{st} \in (a_4(\delta), a_2(\delta))$. First, we show $\underline{\gamma}_s \equiv \underline{\gamma}_s(m_{st}) > 0 = \underline{\gamma}_s(a_4(\delta))$. Suppose not. Then, we must have

$$\begin{split} & \frac{a_4(\delta)^{\sigma}}{\sigma} - \frac{(a_4(\delta) - \bar{s})^{\sigma}}{\sigma} \\ = & \delta \left[\frac{\left[\left[1 - \theta(1 - \beta) \right] \cdot \left[(1 - \beta) + \left[1 - \theta(1 - \beta) \right] \underline{\gamma}_s \right]^{-(1 - \phi)} a_4(\delta) + \theta(1 - \beta) \right]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right] \\ > & \delta \left[\frac{\left[\left[1 - \theta(1 - \beta) \right] \cdot \left[(1 - \beta) + \left[1 - \theta(1 - \beta) \right] \underline{\gamma}_s \right]^{-(1 - \phi)} m_{st} + \theta(1 - \beta) \right]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right] \\ \ge & \frac{m_{st}^{\sigma}}{\sigma} - \frac{(m_{st} - \bar{s})^{\sigma}}{\sigma} \end{split}$$

But it is not possible as $m_{st} > a_4(\delta)$ implies $\frac{m_{st}^{\sigma}}{\sigma} - \frac{(m_{st} - \bar{s})^{\sigma}}{\sigma} > \frac{a_4(\delta)^{\sigma}}{\sigma} - \frac{(a_4(\delta) - \bar{s})^{\sigma}}{\sigma}$. Second, similarly, we can show $\underline{\gamma}_s(m_{st}) < 1 = \underline{\gamma}_s(a_2(\delta))$.

So,
$$\underline{\gamma}_{s}(m_{st}) \in (0,1)$$
 for any $m_{st} \in (a_{4}(\delta), a_{2}(\delta))$.

Third, we show $\underline{\gamma}_s(m_{st})$ must be increasing in m_{st} . Suppose not. $a_4(\delta) < m_{st}^1 < m_{st}^2 < a_2(\delta)$ and $\underline{\gamma}_s^1 \equiv \underline{\gamma}_s(m_{st}^1) \geq \underline{\gamma}_s(m_{st}^2) \equiv \underline{\gamma}_s^2$. Then, we must have

$$\begin{split} & \frac{m_{st}^2}{\sigma} - \frac{(m_{st}^2 - \bar{s})^{\sigma}}{\sigma} \\ = & \delta \left[\frac{\left[\left[1 - \theta(1 - \beta) \right] \cdot \left[(1 - \beta) + \left[1 - \theta(1 - \beta) \right] \underline{\gamma}_s^2 \right]^{-(1 - \phi)} m_{st}^2 + \theta(1 - \beta) \right]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right] \\ > & \delta \left[\frac{\left[\left[1 - \theta(1 - \beta) \right] \cdot \left[(1 - \beta) + \left[1 - \theta(1 - \beta) \right] \underline{\gamma}_s^1 \right]^{-(1 - \phi)} m_{st}^1 + \theta(1 - \beta) \right]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right] \\ = & \frac{m_{st}^1}{\sigma} - \frac{(m_{st}^1 - \bar{s})^{\sigma}}{\sigma} \end{split}$$

But it is not possible as

$$\frac{m_{st}^2}{\sigma} - \frac{(m_{st}^2 - \bar{s})^{\sigma}}{\sigma} < \frac{m_{st}^1}{\sigma} - \frac{(m_{st}^1 - \bar{s})^{\sigma}}{\sigma}.$$

Therefore, when $m_{st} \in (a_4(\delta), a_2(\delta)) \ \underline{\gamma}_s(m_{st}) \in (0, 1)$ and is increasing in m_{st} .

Fourth, we show $\gamma_{st}(m_{st})$ is bounded below by $\underline{\gamma}_s(m_{st})$, i.e.

$$\gamma_{st}(m_{st}) \begin{cases} = \underline{\gamma}_s(m_{st}) & \text{if } \gamma_{ut} = 1, \\ > \underline{\gamma}_s(m_{st}) & \text{if } \gamma_{ut} < 1. \end{cases}$$

We show this in five steps:

Step 1. We show the conjectured mass of skilled worker increases with increase in γ_{ut} , holding γ_{st} constant.

Suppose, $\gamma_{ut}^1 < \gamma_{ut}^2$, then

$$L_{st+1}^{es^1} \equiv \gamma_{ut}^1 \beta (1-\beta) N_{et} + \gamma_{st} p_t^{es} \beta N_{et} < \gamma_{ut}^2 \beta (1-\beta) N_{et} + \gamma_{st} p_t^{es} \beta N_{et} \equiv L_{st+1}^{es^2}$$

Step 2. The conjectured benefit from investment increases with decrease in γ_{ut} , holding γ_{st} constant because

 $\omega_{st+1}^{es^2} < \omega_{st+1}^{es^1}$ where like before $\gamma_{ut}^1 < \gamma_{ut}^2$.

Step 3. Hence, from Step 2 we get the following. If at any m_{st} , $\langle \gamma_{ut}^1, \gamma_{st}^1 \rangle$ and $\langle \gamma_{ut}^2, \gamma_{st}^2 \rangle$ are two equilibria, and like above $\gamma_{ut}^1 < \gamma_{ut}^2$, then

$$\gamma_{st}^1 \ge \gamma_{st}^2$$

where it holds with equality only when $\gamma_{st}^2 = 1$.

Step 4. As γ_{ut} is a probability, the maximum value it can take is 1.

Step 5. Suppose at any $m_{st} \in (a_4(\delta), a_2(\delta)), \langle \gamma_{ut}, \gamma_{st} \rangle$ is an equilibrium. Then by the definition of $\underline{\gamma}_s(m_{st})$ we have

$$\gamma_{st} = \underline{\gamma}_s(m_{st}) \qquad \text{if } \gamma_{ut} = 1$$

and from the argument above $\gamma_{st} > \underline{\gamma}_s(m_{st})$ if $\gamma_{ut} < 1$.

Case 2. Consider any $m_{st} \ge a_2(\delta)$. At $m_{st} = a_2(\delta)$, $\gamma_{st} = 1$ even when $\gamma_{ut} = 1$. So following the argument above, at $m_{st} = a_2(\delta)$, $\gamma_{st} = 1 \ \forall \gamma_{ut} \le 1$. Similarly, we can see, at any $m_{st} > a_2(\delta)$, $\gamma_{st} = 1 \ \forall \gamma_{ut} \le 1$. Hence, at any $m_{st} \ge a_2(\delta)$ $\underline{\gamma}_s(m_{st}) = 1$ and $\gamma_{st}(m_{st}) = \underline{\gamma}_s(m_{st})$.

Case 3. Suppose $m_{st} = a_4(\delta)$. Now it is clear that

$$\gamma_{st}(a_4(\delta)) \begin{cases} = 0 = \underline{\gamma}_s(a_4(\delta)) & \text{if } \gamma_{ut} = 1, \\ > 0 & \text{if } \gamma_{ut} < 1. \end{cases}$$

and that $\underline{\gamma}_s(m_{st})$ is increasing in m_{st} at $a_4(\delta)$.

Condition $\overline{\Gamma}_s$: First, we define $\overline{\gamma}_s(m_{st})$. Then, we prove that

$$\bar{\gamma}_s(m_{st}) \begin{cases} = 0 & \text{at } m_{st} \leq \underline{a}_s(\delta) \\ \in (0,1) & \forall m_{st} \in (\underline{a}_s(\delta), a_6(\delta)) \\ = 1 & \text{at } m_{st} = a_6(\delta). \end{cases}$$

and $\bar{\gamma}_s(m_{st})$ is strictly increasing $\forall m_{st} \in [\underline{a}_s(\delta), a_6(\delta))$. We, also, show that when $m_{st} \in [\underline{a}_s(\delta), a_6(\delta)]$, at any equilibrium γ_{st} is bounded above by $\bar{\gamma}_s(m_{st})$. We consider three cases – Case 1. $m_{st} \in (\underline{a}_s(\delta), a_6(\delta))$, Case 2. $m_{st} = a_6(\delta)$ and Case 3. $m_{st} \leq \underline{a}_s(\delta)$, one by one, and prove the relevant properties of that range.

 $\bar{\gamma}_s(m_{st})$ is defined as follows. A skilled worker is indifferent between investing and not investing when no Educated & Unskilled worker invests and all other Skilled workers invest with probability $\bar{\gamma}_s(m_{st})$.

From the definitions of $\underline{a}_s(\delta)$ and $a_6(\delta)$, now it is clear that

at
$$m_{st} = \underline{a}_s(\delta)$$
 $\bar{\gamma}_s(m_{st}) = 0$, and at $m_{st} = a_6(\delta)$, $\bar{\gamma}_s(m_{st}) = 1$.
Case 1. Suppose $m_{st} \in (\underline{a}_s(\delta), a_6(\delta))$

First, following the argument used in Condition $\underline{\Gamma}_s$ above, it can be shown that, at this range of m_{st} , $\overline{\gamma}_s(m_{st}) \in (0, 1)$ and is increasing in m_{st} .

Second, we show that, at this range of m_{st} , $\gamma_{st}(m_{st})$ is bounded above by $\bar{\gamma}_s(m_{st})$, i.e.

$$\gamma_{st}(m_{st}) \begin{cases} = \bar{\gamma}_s(m_{st}) & \text{if } \gamma_{ut} = 0, \\ < \bar{\gamma}_s(m_{st}) & \text{if } \gamma_{ut} > 0 \end{cases}.$$

Following the argument used in Condition $\underline{\Gamma}_s$ above, we can show that if at any m_{st} , $\langle \gamma_{ut}^1, \gamma_{st}^1 \rangle$ and $\langle \gamma_{ut}^2, \gamma_{st}^2 \rangle$ are two equilibria, and $\gamma_{ut}^1 < \gamma_{ut}^2$, then

$$\gamma_{st}^1 \ge \gamma_{st}^2$$

where it holds with equality only when $\gamma_{st}^2 = 1$.

Now, as γ_{ut} is a probability, the minimum value it can take is 0.

Suppose, at any $m_{st} \in (\underline{a}_s(\delta), a_6(\delta)), \langle \gamma_{ut}, \gamma_{st} \rangle$ is an equilibrium. Then by the definition of $\overline{\gamma}_s(m_{st})$ we have

$$\gamma_{st} = \bar{\gamma}_s(m_{st}) \quad \text{ if } \gamma_{ut} = 0$$

and from the argument above

$$\gamma_{st} < \bar{\gamma}_s(m_{st}) \quad \text{ if } \gamma_{ut} > 0.$$

Case 2. Suppose $m_{st} = a_6(\delta)$. Now it is clear that at any equilibrium $\langle \gamma_{ut}, \gamma_{st} \rangle$

$$\gamma_{st}(a_6(\delta)) \begin{cases} = 1 = \bar{\gamma}_s(a_6(\delta)) & \text{if } \gamma_{ut} = 0, \\ < 1 & \text{if } \gamma_{ut} > 0. \end{cases}$$

Case 3. Suppose $m_{st} \leq \underline{a}_s(\delta)$. By the definition of $\underline{a}_s(\delta)$ and $\overline{\gamma}_s$, we have $\overline{\gamma}_s(m_{st}) = 0$.

Thus, at any equilibrium $\langle \gamma_{ut}, \gamma_{st} \rangle$, $\gamma_{st} = 0 = \bar{\gamma}_s$. And also observe, $\bar{\gamma}_s(m_{st})$ is increasing in m_{st} at $m_{st} = \underline{a}_s(\delta)$.

(iii) Can be proved following the argument used in the proof of Condition $\underline{\Gamma}_s$.

(iv) Can be proved following the argument used in the proof of Condition $\overline{\Gamma}_s$.

C.6 Characterization of Equilibria

We introduce an observation and a lemma which we use to prove Proposition 5 in C.6.1.

Observation C.1. Suppose there are two equilibria $\langle \gamma_{ut}, \gamma_{st} \rangle$ and $\langle \tilde{\gamma}_{ut}, \tilde{\gamma}_{st} \rangle$ at any $m_{st} \geq 1$. If $\gamma_{jt} < \tilde{\gamma}_{jt}$ then $\tilde{\gamma}_{kt} \leq \gamma_{kt}$ where $j, k \in \{u, s\}$ and $j \neq k$. The latter inequality binds only when $\tilde{\gamma}_{kt} = 1$.

Proof. Immediate from investment decisions of both types of parents given by (5) and (6). \Box

Lemma C.1. Suppose $\delta \in (\underline{\delta}, \delta_a]$.

- (i) Suppose $\theta(1-\beta) \neq \beta$, then at any $m_{st} \in [1, \min\{a_1, a_2\})$, there can be at most one equilibrium where both types of workers play mixed strategies.
- (ii) At any $m_{st} \in [1, \min\{a_1(\delta), a_2(\delta)\})$, there can be multiple equilibria only if $\beta \ge \theta(1-\beta)$.
- (iii) Suppose $\beta < \theta(1-\beta)$. Let $\langle \gamma_{ut}, \gamma_{st} \rangle$ be an equilibrium at any $m_{st} \in [1, \min\{a_1(\delta), a_2(\delta)\})$. If $\gamma_{st} > \gamma_{ut}$, then at all $\tilde{m}_{st} \in (m_{st}, \min\{a_1(\delta), a_2(\delta)\})$ $\tilde{\gamma}_{st} > \tilde{\gamma}_{ut}$ where $\langle \tilde{\gamma}_{ut}, \tilde{\gamma}_{st} \rangle$ is an equilibrium at \tilde{m}_{st} .

Proof. $\delta \in (\underline{\delta}, \delta_a]$. Then from Lemma 2 and Lemma 4, we have $\underline{a}_s(\delta) < 1$.

(i) Suppose not. $\theta(1-\beta) \neq \beta$ and at some $m_{st} \in [1, \min\{a_1, a_2\})$, there exist two equilibria $\langle \gamma_{ut}, \gamma_{st} \rangle \langle \tilde{\gamma}_{ut}, \tilde{\gamma}_{st} \rangle$ where both types of workers play mixed strategies, i.e. $0 < \gamma_{ut} \neq \tilde{\gamma}_{ut} < 1$ and $0 < \gamma_{st} \neq \tilde{\gamma}_{st} < 1$.

Then from the decision of Educated & Unskilled workers, given by (5), we must have

$$\theta(1-\beta)\gamma_{ut} + \beta\gamma_{st} = \theta(1-\beta)\tilde{\gamma}_{ut} + \beta\tilde{\gamma}_{st} \tag{A.15}$$

And, from the decision of skilled workers, given by (6), we must have

$$(1-\beta)\gamma_{ut} + [1-\theta(1-\beta)]\gamma_{st} = (1-\beta)\tilde{\gamma}_{ut} + [1-\theta(1-\beta)]\tilde{\gamma}_{st}$$
(A.16)

As $\theta(1-\beta) \neq \beta$, from (A.15) and (A.16), we have $\gamma_{ut} = \tilde{\gamma}_{ut}$ and $\gamma_{st} = \tilde{\gamma}_{st}$ – a contradiction.

(ii) We find necessary condition for the existence of multiple equilibria at any $m_{st} \in [1, \min\{a_1(\delta), a_2(\delta)\})$. Given Definition of $a_1(\delta), a_2(\delta)$, Lemma 3, and part (i) of this lemma, at any $m_{st} \in [1, \min\{a_1(\delta), a_2(\delta)\})$, if there are two equilibria $\langle \gamma_{ut}, \gamma_{st} \rangle$ and $\langle \tilde{\gamma}_{ut}, \tilde{\gamma}_{st} \rangle$, then we must have

$$1 \ge \gamma_{ut} > \tilde{\gamma}_{ut} \ge 0$$
 and $1 \ge \tilde{\gamma}_{st} \ge \gamma_{st} > 0$.

Due to (5) and (6), we must have

$$\theta(1-\beta)\tilde{\gamma}_{ut} + \beta\tilde{\gamma}_{st} \ge \theta(1-\beta)\gamma_{ut} + \beta\gamma_{st}$$

$$\Rightarrow \quad \beta[\tilde{\gamma}_{st} - \gamma_{st}] \ge \theta(1-\beta)[\gamma_{ut} - \tilde{\gamma}_{ut}]$$
(A.17)

$$(1-\beta)\gamma_{ut} + [1-\theta(1-\beta)]\gamma_{st} \ge (1-\beta)\tilde{\gamma}_{ut} + [1-\theta(1-\beta)]\tilde{\gamma}_{st}$$

$$\Rightarrow \quad (1-\beta)[\gamma_{ut} - \tilde{\gamma}_{ut}] \ge [1-\theta(1-\beta)][\tilde{\gamma}_{st} - \gamma_{st}]. \tag{A.18}$$

Hence, combining (A.17) and (A.18), we get the necessary condition for the coexistence of $\langle \gamma_{ut}, \gamma_{st} \rangle$ and $\langle \tilde{\gamma}_{ut}, \tilde{\gamma}_{st} \rangle$:

$$\beta \ge \theta [1 - \theta (1 - \beta)] \qquad \Rightarrow \quad \beta \ge \theta (1 - \beta).$$

(iii) Suppose not. $\gamma_{st} > \gamma_{ut}$ and $\exists \tilde{m}_{st} > m_{st}$ such that $\tilde{\gamma}_{ut} \ge \tilde{\gamma}_{st}$.

First observe, as $\beta < \theta(1-\beta)$, at any m_{st} there is a unique equilibrium $\langle \gamma_{ut}, \gamma_{st} \rangle$. Second, both m_{st} and \tilde{m}_{st} less than $min\{a_1(\delta), a_2(\delta)\}$ implies $\gamma_{ut} < 1$ and $\tilde{\gamma}_{st} < 1$.

Now from the investment decision of Educated & Unskilled workers, given by (5), we have

$$\frac{\theta \gamma_{ut}(1-\beta) + \beta \gamma_{st}}{\theta \tilde{\gamma}_{ut}(1-\beta) + \beta \tilde{\gamma}_{st}} \ge \left[\frac{\tilde{m}_{st}}{m_{st}}\right]^{-\frac{1}{1-\phi}}$$

And from the investment decision of Skilled workers, given by (6), we have

$$\frac{(1-\beta)\gamma_{ut} + [1-\theta(1-\beta)]\gamma_{st}}{(1-\beta)\tilde{\gamma}_{ut} + [1-\theta(1-\beta)]\tilde{\gamma}_{st}} < \left[\frac{\tilde{m}_{st}}{m_{st}}\right]^{-\frac{1}{1-\phi}}$$

From these two conditions we get

$$\frac{\theta \gamma_{ut}(1-\beta) + \beta \gamma_{st}}{\theta \tilde{\gamma}_{ut}(1-\beta) + \beta \tilde{\gamma}_{st}} > \frac{(1-\beta)\gamma_{ut} + [1-\theta(1-\beta)]\gamma_{st}}{(1-\beta)\tilde{\gamma}_{ut} + [1-\theta(1-\beta)]\tilde{\gamma}_{st}}$$

$$\Rightarrow \quad [\tilde{\gamma}_{ut}\gamma_{st} - \gamma_{ut}\tilde{\gamma}_{st}][\beta - \theta[1-\theta(1-\beta)]] > 0$$

$$\Rightarrow \quad \beta - \theta[1-\theta(1-\beta)] > 0$$

$$\Rightarrow \quad \beta > \theta(1-\beta).$$

the second last line follows from $\gamma_{st} > \gamma_{ut}$ and $\tilde{\gamma}_{ut} \ge \tilde{\gamma}_{st}$. A contradiction as $\beta < \theta(1-\beta)$. \Box

C.6.1 Proof of Proposition 5

- 1. As $\eta = 0$, this is trivial.
- 2. $\delta > \delta_a$, so from Lemma 4. iii. (b), we know $a_1(\delta) < 1$. So, by the definition of $a_1(\delta)$ $\gamma_{ut} = 1 \ \forall m_{st} \ge 1$.

Now, from Lemma 4. iv., we know $a_4(\delta) < 1$. So, due to Lemma 5. Condition $\underline{\Gamma}_s$, γ_{st} must be no less than $\underline{\gamma}_s(m_{st}) \ \forall m_{st} \geq 1$.

The equilibrium is unique as at any $m_{st} \ge 1$, $\gamma_{ut} = 1$. This implies equilibrium γ_{st} would be equal to $\gamma_{o}(m_{st})$ which is unique.

3. (i) If $a_2(\delta) \ge a_1(\delta)$, by the definition $a_1(\delta)$ we have $\gamma_{ut} = 1 \ \forall \gamma_{st} \in [0, 1]$. Now, by Lemma 4. iv., we have $a_4(\delta) \le a_1(\delta)$. Hence, due to Lemma 5 Condition $\underline{\Gamma}_s$, γ_{st} must be no less than $\underline{\gamma}_s(m_{st}) \ \forall m_{st} \ge a_1(\delta)$. The equilibrium is unique as at any $m_{st} \ge a_1(\delta)$, $\gamma_{ut} = 1$. This implies equilibrium γ_{st} would be equal to $\underline{\gamma}_s(m_{st})$ which is unique.

If $a_2(\delta) < a_1(\delta)$, by the definition $a_2(\delta)$ we have $\gamma_{st} = 1 \ \forall \gamma_{ut} \in [0, 1]$. By the definition of $a_5(\delta)$, (5), and Lemma 5 Condition $\underline{\Gamma}_u$, we have that

 $\gamma_{ut} = 0 \quad \forall m_{st} \in [a_2(\delta), a_5(\delta)) \qquad \gamma_{ut} = \underline{\gamma}_u(m_{st}) \quad \forall m_{st} \ge a_5(\delta).$

That the equilibrium is unique is now evident.

- (ii) Due to Lemma 5 Condition $\underline{\Gamma}_s$, Condition $\overline{\Gamma}_s$, Condition $\underline{\Gamma}_u$ and Condition $\overline{\Gamma}_u$ must be satisfied whenever possible.
 - (a) This follows from Part (iii) of Lemma C.1.
 - (b) This follows from Part (i) and (ii) of Lemma C.1.
- 4. From Lemma 4, we have $a_3(\delta), a_5(\delta), a_1(\delta)$ tend to infinity. Hence, the Educated & Unskilled workers do not invest for any finite state variable: $\gamma_{ut} = 0 \ \forall \gamma_{st} \in [0, 1]$.

From Lemma 2. (iii) and Lemma 4. v. (a) and ii. (b), we have

$$1 < \underline{b}_s(\delta) = \underline{a}_s(\delta) < a_6(\delta).$$

So, due to Lemma 5. Condition $\underline{\Gamma}_s$, for any $m_{st} \in [1, a_6(\delta)], \gamma_{st} \leq \overline{\gamma}_s(m_{st})$.

The equilibrium is unique as $\gamma_{ut} = 0$ for any finite m_{st} . Hence, by Condition $\underline{\Gamma}_s$ depicted in Lemma 5., we must have $\gamma_{st} = \overline{\gamma}_s(m_{st}) \ \forall m_{st} \in [1, a_6(\delta)]$ and by definition of $a_6(\delta)$, $\gamma_{st} = 1 \ \forall m_{st} > a_6(\delta)$.

C.7 Proof of Proposition 6

1. Suppose an economy with degree of child affinity not low starts with all educated adults. Then the income of a skilled worker that is the state variable would be $A\beta^{-(1-\phi)}$. So, if $A\beta^{-(1-\phi)} \ge \max\{a_1(\delta), a_2(\delta)\}$. Then all parents would invest at all t and there would be no poverty trap.

If $A\beta^{-(1-\phi)} < \max\{a_1(\delta), a_2(\delta)\}$ or the economy starts with a positive mass of Not Educated adults, then there will be a positive mass of Not Educated workers from t = 1onwards. We have seen that Not Educated workers never invest and education is necessary for getting a skilled job. Hence, the mass of families which never become rich is positive.

When the degree of child affinity is low, then no unskilled worker invests, so there would be a poverty trap in the economy. Thus, in any economy, there exist a poverty trap almost always.

2. (d) We note that both $a_1(\delta)$ and $a_2(\delta)$ are decreasing in δ , so the max $\{a_1(\delta), a_2(\delta)\}$ is also decreasing in δ . Now, the inequality at the 'least unequal steady state' remains constant for all δ no less than the particular degree of child affinity where max $\{a_1(\delta), a_2(\delta)\} = A\beta^{-(1-\phi)}$. As in at such a δ , at the 'least unequal steady state' all parents invest.

Such a δ exists $\forall \theta > 0$, as the benefit from investment for both types of parents increase with δ and δ can be very large.

The rest of the proof is very similar to the proof of Proposition 4, so we skip it. \Box

D Appendix for Comparison

D.1 Proof of Observation 8

1. (A) It is obvious as $b_s(\delta)$ and $\underline{b}_s(\delta)$ are the same in the benchmark case and in the case with behavioral trap. Moreover, at any m_{st} , $\rho_{ut} > 0$, $\rho_{st} = \lambda_{st} = 1$.

1. (B) From Proposition 3, we have that when $\delta \geq \overline{\delta}$ then $\rho_{ut} = 1$.

Now, we argue when $\delta \in (\underline{\delta}, \overline{\delta})$, the mass of Not Educated workers is positive and $m_{st} > \underline{b}_u(\delta)$ then $\rho_{ut} > \lambda_{ut} > 0$.

Lemma 2 implies $\underline{b}_u(\delta) > \overline{b}_s(\delta)$. So, $\lambda_{st} = \rho_{st} = 1$ and $\lambda_{ut} \in (0, 1)$. Then, from (3) and (4)

$$\delta \left[\frac{\left[\beta^{\phi}A[\rho_{ut}(1-\beta)N_{et}+\beta N_{et}]^{-(1-\phi)}+1-\beta\right]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right]$$

$$\geq \frac{1}{\sigma} - \frac{(1-\bar{s})^{\sigma}}{\sigma}$$

$$= \delta \left[\frac{\left[\beta^{\phi}A[\lambda_{ut}[(1-\beta)N_{et}+(1-N_{et})]+\beta N_{et}]^{-(1-\phi)}+1-\beta\right]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right]$$

$$\Rightarrow \rho_{ut}(1-\beta)N_{et} \geq \lambda_{ut}[(1-\beta)N_{et}+(1-N_{et})] > \lambda_{ut}(1-\beta)N_{et}$$

$$\Rightarrow \rho_{ut} > \lambda_{ut}.$$

where the first inequality comes from $\rho_{ut} > 0$, it would bind if $\rho < 1$, and we get the second last inequality, i.e. $\lambda_{ut}[(1 - \beta)N_{et} + (1 - N_{et})] > \lambda_{ut}(1 - \beta)N_{et}$ because the mass of Not Educated workers is positive, i.e. $1 - N_{et} > 0$ and $\lambda_{ut} > 0$.

This is now immediate that when $\delta \in (\underline{\delta}, \overline{\delta})$, the mass of Not Educated workers is zero and $m_{st} > \underline{b}_u(\delta)$ then $\rho_{ut} = \lambda_{ut} > 0$.

Finally, from the definition of $\underline{b}_u(\delta)$, when $\delta \in (\underline{\delta}, \overline{\delta})$, and $m_{st} \leq \underline{b}_u(\delta)$ then $\rho_{ut} = \lambda_{ut} = 0$.

2. This is immediate as at $\delta < \underline{\delta}$, $\lambda_{ut} = \rho_{ut} = 0$ and $\lambda_{st} = \rho_{st}$.

D.2 Proof of Observation 9

From Proposition 2 and Proposition 4 we see that when the degree of child affinity is not low, then the inequality at the unique steady state of the benchmark case is equal to the inequality at the least unequal steady state of the case with behavioral trap. At any other steady state inequality is higher. Hence, the result.

D.3 Proof of Observation 10

We have already noted in Observation 2 that $\underline{\delta} < \overline{\delta}$. Now we show

$$\bar{\delta} \equiv \frac{(1-\bar{s})^{\sigma} - 1}{1 - [A\beta^{\phi} + 1 - \beta]^{\sigma}} < \frac{(1-\bar{s})^{\sigma} - 1}{1 - [\theta\beta(\theta(1-\beta) + \beta)^{-(1-\phi)} + 1 - \theta\beta]^{\sigma}} \equiv \delta_a.$$

Comparing these δ values we get that this statement is true if and only if

$$\theta\beta(\theta(1-\beta)+\beta)^{-(1-\phi)}+1-\theta\beta < A\beta^{\phi}+1-\beta$$
(A.19)

We define a function $L(\theta)$ and derive its properties:

$$\begin{split} L(\theta) &= \theta(\theta(1-\beta)+\beta)^{-(1-\phi)} - \theta + 1 - \beta^{-(1-\phi)} \quad \text{and} \ L(0) = L(1) = 1 - \beta^{-(1-\phi)} < 0 \\ L'(\theta) &= (\beta + \theta(1-\beta))^{-(2-\phi)}(\beta + \phi\theta(1-\beta)) - 1 \\ L'(0) &= \beta^{-(1-\phi)} - 1 > 0 \quad \text{and} \ L'(1) = -(1-\beta)(1-\phi) < 0 \\ L'(\bar{\theta}) &= 0 \quad \text{where} \quad \bar{\theta} \ : (\beta + \bar{\theta}(1-\beta))^{-(1-\phi)} = \frac{\beta + \bar{\theta}(1-\beta)}{\beta + \phi\bar{\theta}(1-\beta)} < \beta^{-(1-\phi)} \\ L(\bar{\theta}) &= \underbrace{\frac{\beta + \bar{\theta}(1-\beta)(\phi + \bar{\theta}(1-\phi))}{\beta + \bar{\theta}(1-\beta)}}_{fraction} \underbrace{(\beta + \bar{\theta}(1-\beta))^{-(1-\phi)}}_{<\beta^{-(1-\phi)}} - \beta^{-(1-\phi)} < 0 \\ L''(\theta) &= -(1-\phi)(1-\beta)(\beta + \theta(1-\beta))^{-(3-\phi)}(2\beta + \theta\phi(1-\beta)) < 0 \end{split}$$

Thus L is (a) a strictly concave function, (b) has a maxima at $\bar{\theta}$, (c) the maximum value is negative. Thus, $L(\theta)$ is negative values for all $\theta \in [0, 1]$. Thus,

$$\begin{aligned} \theta(\theta(1-\beta)+\beta)^{-(1-\phi)} &-\theta+1-\beta^{-(1-\phi)} < 0\\ \Rightarrow \quad \theta\beta(\theta(1-\beta)+\beta)^{-(1-\phi)}+1-\theta\beta < \beta^{\phi}+1-\beta < A\beta^{\phi}+1-\beta \end{aligned}$$

Thus, equation (A.19) always holds and hence $\bar{\delta} < \delta_a$.

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Supplementary Appendix

7.4 Equilibria Strategies

Given Lemma 1 (a), there can be five equilibria:

- χ 1. Both unskilled and skilled workers invest with certainty: $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle 1, 1 \rangle$.
- χ^2 . Unskilled workers invest with positive probability and skilled workers invest with certainty: $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle (0, 1), 1 \rangle$.
- χ 3. Unskilled workers do not invest and skilled workers invest with certainty: $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle 0, 1 \rangle$.
- $\chi 4$. Unskilled workers do not invest and skilled workers invest with positive probability: $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle 0, (0, 1) \rangle.$
- χ 5. Both unskilled and skilled workers do not invest: $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle 0, 0 \rangle$.

We consider all the strategies below. Recall the notation: L_{ut} : Mass of unskilled workers at period t, L_{st} : Mass of skilled workers at period t, N_{et} : Mass of educated adult at period t.

 χ **1.** Both unskilled and skilled workers invest with certainty: $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle 1, 1 \rangle$ All skilled and unskilled parents invest in their children's education. So,

$$N_{et+1} = 1$$

$$L_{st+1} = \beta (L_{st} + L_{ut}) = \beta, \qquad m_{st+1} = AL_{st+1}^{-(1-\phi)} = A\beta^{-(1-\phi)}$$

$$L_{ut+1} = 1 - L_{st+1} = 1 - \beta, \qquad m_{ut+1} = 1.$$

As observed in Lemma 1.(a), whenever an unskilled worker has an incentive to invest, a skilled worker also has an inventive to invest with certainty. So, this case $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle 1, 1 \rangle$ arises when the parametric condition is such that an unskilled worker has an incentive to invest with certainty, when all other workers invest with certainty. We, now, characterize such parametric condition.

At any t, both unskilled and skilled workers invest with certainty is an equilibrium if and only if

$$\delta \left[\frac{[\beta m_{st+1} + (1-\beta)m_{ut+1}]^{\sigma}}{\sigma} - \frac{1}{\sigma} \right] \ge \frac{1}{\sigma} - \frac{(1-\bar{s})^{\sigma}}{\sigma}$$
$$\Rightarrow \quad \delta \ge \frac{(1-\bar{s})^{\sigma} - 1}{1 - [A\beta^{\phi} + 1 - \beta]^{\sigma}} \equiv \bar{\delta} \tag{A.20}$$

So, when the parents are very child loving, all workers invest with certainty.

Observe, this condition is time independent, hence if this condition satisfies at certain t, then it holds at all other t. This implies that the economy enters in to a steady state at t = 2, we summarize this in the following observation.

Observation 7.1. If and only if the degree of child affinity is high, i.e. $\delta \geq \overline{\delta}$

(i) All parents always invest with certainty.



Figure 7: If and only if $\delta \geq \overline{\delta}$, $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle 1, 1 \rangle$

(ii) At t = 2, the economy immediately enters into a steady state.

At the steady state, all types of parents invest with certainty. Hence, the number of skilled and unskilled workers and their respective incomes remain constant over time.

At steady state: the mass of educated individual $N_e^* = 1$, the mass of skilled workers $L_s^* = \beta$, and wage of a skilled worker $m_s^* = A\beta^{-(1-\phi)}$, the mass of unskilled workers $L_u^* = 1 - \beta$, and wage of an unskilled worker $m_s^* = 1$.

χ 2. Unskilled workers invest with positive probability: $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle (0, 1), 1 \rangle$

Unskilled workers invest with positive probability $\lambda_{ut} \in (0, 1)$ and skilled workers invest with certainty. So,

$$N_{et+1} = \lambda_{ut} L_{ut} + L_{st}$$
$$L_{st+1} = \beta(\lambda_{ut} L_{ut} + L_{st}), \qquad m_{st+1} = A \left(\beta(\lambda_{ut} L_{ut} + L_{st})\right)^{-(1-\phi)}$$
$$L_{ut+1} = (1 - \beta\lambda_{ut})L_{ut} + (1 - \beta)L_{st}, \qquad m_{ut+1} = 1.$$

We are looking for parametric condition such that an unskilled worker is indifferent between investing and not investing when all other unskilled workers are investing with probability $\lambda_{ut} \in (0, 1)$ and all skilled workers are investing with certainty. Whenever the degree of child affinity is low i.e. $\delta < \bar{\delta}$ unskilled parents invest with probability strictly less than 1. They invest with positive probability only when m_{st} is higher than $\underline{b}_u(\delta)$. From Lemma 2 we know $\underline{b}_u(\delta)$ is infinite when the degree of child affinity is low, i.e. $\delta < \underline{\delta}$. This gives us the part (a) of the following observation.

Now, coming to the dynamics, we show that λ_{ut} is such that the economy enters into a steady state at t + 1. For that consider, again, the incentive constraint of an unskilled worker when when all other unskilled workers are investing with probability λ_{ut} and all skilled workers are investing with certainty

$$\frac{(m_{ut}-\bar{s})^{\sigma}}{\sigma} + \delta \frac{[\beta m_{st+1} + (1-\beta)m_{ut+1}]^{\sigma}}{\sigma} = \frac{m_{ut}^{\sigma}}{\sigma} + \delta \frac{m_{ut+1}^{\sigma}}{\sigma}$$

At the steady state, $L_{st+1} = L_{st}$ which implies

$$\beta(\lambda_{ut} + (1 - \lambda_{ut})L_{st}) = L_{st+1} = \left[\frac{1}{\beta A} \left[\left[\frac{1 + \delta - (1 - \bar{s})^{\sigma}}{\delta}\right]^{\frac{1}{\sigma}} - (1 - \beta) \right] \right]^{-\frac{1}{1 - \phi}} \equiv \beta \left(\frac{\underline{b}_u(\delta)}{A}\right)^{-\frac{1}{1 - \phi}}$$
$$\Rightarrow \quad \lambda_{ut} = \frac{\left(\underline{b}_u(\delta)/A\right)^{-\frac{1}{1 - \phi}} - L_{st}}{1 - L_{st}}$$

Observe, $\underline{b}_u(\delta)$ is time independent, hence L_{st+1} is time independent. So, if an economy is such that $\lambda_{ut} = \frac{(\underline{b}_u(\delta)/A)^{-\frac{1}{1-\phi}} - L_{st}}{1 - L_{st}}$ and $\lambda_{st} = 1$, then the economy is at a steady state at t+1. At the steady state, mass of skilled worker $L_s^* \equiv \beta (\underline{b}_u(\delta)/A)^{-\frac{1}{1-\phi}}$, wage of a skilled worker $m_s^* \equiv \beta^{-(1-\phi)}\underline{b}_u(\delta)$ and $\lambda_u^* \equiv \frac{(\underline{b}_u(\delta)/A)^{-\frac{1}{1-\phi}} - L_s^*}{1 - L_s^*}$.

To check that indeed this is a steady state, we need $\lambda_u^* \in (0,1)$. Now, $\lambda_u^* < 1$ because the degree of child affinity is not high, i.e. $\delta < \overline{\delta}$. And, $\lambda_u^* > 0$ because $m_s^* \equiv \beta^{-(1-\phi)} \underline{b}_u(\delta) > \underline{b}_u(\delta)$. Hence, the part (b) of the following observation.

Observation 7.2. If and only if the degree of child affinity is moderate i.e. $\delta \in [\underline{\delta}, \overline{\delta})$ and $m_{st} \in (\underline{b}_u(\delta), \infty)$, then

(a) unskilled workers invest with probability λ_{ut} , where $\lambda_{ut} \in (0, 1)$, and all skilled workers invest with certainty is the unique equilibrium.



Figure 8: At period t, $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle (0,1), 1 \rangle$ is an equilibrium

(b) The economy enters into a steady state at t + 1. At steady state,

- the mass of skilled worker is $L_s^* \equiv \beta \left(\underline{b}_u(\delta) / A \right)^{-\frac{1}{1-\phi}}$,
- the wage of a skilled worker $m_s^* \equiv \beta^{-(1-\phi)} \underline{b}_u(\delta)$
- and an unskilled worker invests with probability $\lambda_u^* \equiv \frac{\left(\underline{b}_u(\delta)/A\right)^{-\frac{1}{1-\phi}} L_s^*}{1 L_s^*}.$

 χ 3. Unskilled workers do not invest and skilled workers invest with certainty: $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle 0, 1 \rangle$

All skilled parents invest with probability 1 and no unskilled parents invest. So,

$$N_{et+1} = L_{st}$$

$$L_{st+1} = \beta L_{st}, \qquad m_{st+1} = AL_{st+1}^{-(1-\phi)} = \beta^{-(1-\phi)}m_{st}$$

$$L_{ut+1} = 1 - L_{st+1}, \qquad m_{ut+1} = 1.$$

It follows from the definition of \bar{b}_s that the income at which skilled workers invest with unit probability is $m_{st} > \bar{b}_s$. Similarly, income range for which unskilled parents do not invest with certainty is $m_{st} < \underline{b}_u$. Now, $\underline{b}_u(\delta)$ is infinite when $\delta < \underline{\delta}$. This gives us the part (a) of the observation. Now, at this parametric condition, all skilled workers invest with probability 1 and no unskilled workers invest, so in the next period, the mass of educated individual would be equal to the mass of skilled workers at this period. Among those educated individuals β part will become skilled workers. Hence, the part (b) of the following observation.

Observation 7.3. (i) At any period t, all skilled workers invest with certainty and no unskilled workers invest if

- (a) either the degree of child affinity is moderate, i.e. $\delta \in [\underline{\delta}, \overline{\delta})$ and $m_{st} \in [\overline{b}_s, \underline{b}_u)$
- (b) or the degree of child affinity is low, i.e. $\delta \in (0, \underline{\delta})$ and $m_{st} \in [\overline{b}_s, \infty)$.



Figure 9: At period t, $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle 0, 1 \rangle$ is an equilibrium

- (ii) If either of the above two conditions is satisfied, then the number of skilled workers fall at the rate β while skilled income rises at the rate $\beta^{-(1-\phi)}$.
- χ 4. Unskilled workers do not invest and skilled invest with positive probability: $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle 0, (0, 1) \rangle$

Unskilled workers do not invest and skilled workers invest with probability $\lambda_{st} \in (0, 1)$. So,

$$N_{et+1} = \lambda_{st} L_{st}$$
$$L_{st+1} = \beta \lambda_{st} L_{st}, \qquad m_{st+1} = A \left(\beta \lambda_{st} L_{st}\right)^{-(1-\phi)} = \left(\beta \lambda_{st}\right)^{-(1-\phi)} m_{st}$$
$$L_{ut+1} = 1 - \beta \lambda_{st} L_{st}, \qquad m_{ut+1} = 1.$$

We are looking for parametric condition such that a skilled worker is indifferent between investing and not investing when all other skilled workers are investing with probability λ_{st} and no unskilled workers are investing. Hence, m_{st} must be higher than $\underline{b}_s(\delta)$ but lower than $\overline{b}_s(\delta)$. Now, in Lemma 2, we have seen that $\underline{b}_s(\delta) > 1$ if and only if $\delta < \underline{\delta}$ and in Lemma 1, we have seen that $m_{st} \geq 1$. This gives us the part (a) of the observation. Now, at this parametric condition, skilled workers invest with probability λ_{st} and no unskilled workers invest, so in the next period, the mass of educated individual would be less than the mass of skilled workers at this period. Among those educated individuals β part will become skilled workers. Hence, the part (b) of the following observation.

Observation 7.4. (i) At any period t, no unskilled workers invest and skilled workers invest with probability λ_{st} such that $\lambda_{st} \in (0, 1)$ and if

- (a) either the degree of child affinity is moderate, i.e. $\delta \in [\underline{\delta}, \overline{\delta})$ and $m_{st} \in [1, \overline{b}_s(\delta))$
- (b) or the degree of child affinity is low, i.e. $\delta \in (0, \underline{\delta})$ and $m_{st} \in (\underline{b}_s(\delta), \overline{b}_s(\delta))$.



Figure 10: At period t, $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle 0, (0, 1) \rangle$ is an equilibrium

(ii) If either of the above two conditions is satisfied, then the number of skilled workers fall and the income of skilled workers rises over time.

χ 5. Both unskilled workers and skilled workers do not invest: $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle 0, 0 \rangle$

No worker invests, so

$$N_{et+1} = 0$$

 $L_{st+1} = 0, \qquad m_{st+1} \to \infty$
 $L_{ut+1} = 1, \qquad m_{ut+1} = 1.$

It follows from the definition of $\underline{b}_s(\delta)$ that m_{st} must be lower than $\underline{b}_s(\delta)$. Now, in Lemma 2, we have seen that $\underline{b}_s(\delta) > 1$ if and only if $\delta < \underline{\delta}$. Together with Lemma 1, we find that neither skilled nor unskilled parents invest in their children education for $\delta < \underline{\delta}$ and $1 \le m_{st} < \underline{b}_s$ – explains part (a) of the following observation. Now, we have observed that a skilled worker does not have any incentive to invest when no other worker is investing. In the next period, thus, all workers would be unskilled and income of an unskilled worker is lower than that of a skilled worker, so from the next period onwards also, no parent would ever invest in her child's education. Hence, the part (b) of the following observation. **Observation 7.5.** If and only if degree of child affinity is low $\delta < \underline{\delta}$ and $m_{st} \in [1, \underline{b}_s(\delta))$

(i) No workers invest.



Figure 11: At period t, $\langle \lambda_{ut}, \lambda_{st} \rangle = \langle 0, 0 \rangle$ is an equilibrium

(ii) The economy immediately jumps to steady state with no skilled or educated worker. Further as $m_{st+1} \rightarrow \infty$.



Figure 12: Condition $\underline{\Gamma}_u$ and $\overline{\Gamma}_u$

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University Enclave, University of Delhi (North Campus) Delhi 110007, India Tel: 27667288/365/424 Email: system@iegindia.org